Common Core Progress™

Mathematics

4

Sadlier® School

Common Core Progress™

Mathematics

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Welcome

You have an exciting year ahead of you. You will be learning more about mathematics and the tools you will need to solve everyday problems.

Did you know that you solve problems and use math all the time? Think about your day. When you play sports after school, shop at your favorite store, cook delicious food, build something awesome like a tree house, or travel in a car, bus or train, you are using math and applying that understanding to make sense of the world around you.

Common Core Progress will help you improve problem-solving skills while becoming more confident in mathematics. That's why it's called *progress*.

Have a great year!

Progress Check

UNIT 1

Look at how the Common Core standards you have learned and will learn connect.

It is very important for you to understand the standards from the prior grade level so that you will be able to develop an understanding of operations and algebraic thinking in this unit and be prepared for next year. To practice your skills, go to sadlierconnect.com.

GRADE 3		GRADE 4		GRADE 5
I Can...	Before Unit 1	**Can I ?**	After Unit 1	**I Will...**
3.OA.1 Interpret products of whole numbers	☐	**4.OA.1** Interpret a multiplication equation as a comparison	☐	**5.OA.1** Write and evaluate expressions that use parentheses, brackets, or braces
	☐	Write multiplication equations to represent comparisons	☐	**5.NF.5** Interpret multiplication as scaling (resizing)
3.OA.3 Multiply and divide within 100 to solve word problems	☐	**4.OA.2** Solve word problems involving multiplicative comparisons by multiplying or dividing	☐	**5.OA.2** Write simple expressions that record calculations with numbers
3.OA.8 Solve two-step word problems using the four operations	☐	**4.OA.3** Solve multistep word problems involving whole numbers using the four operations	☐	
Represent two-step problems using equations	☐	Represent multistep problems using equations	☐	
3.OA.6 Find unknown factors to solve division problems	☐	**4.OA.4** Find all factor pairs for a whole number from 1 to 100	☐	
	☐	Determine if a whole number from 1 to 100 is a multiple of a given one-digit number	☐	
	☐	Determine if a number from 1 to 100 is prime or composite	☐	
3.OA.9 Identify arithmetic patterns	☐	**4.OA.5** Generate a pattern that follows a given rule	☐	**5.OA.3** Generate two numerical patterns using two given rules, and explore how the two patterns are related
	☐	Explore and explain features of patterns	☐	

HOME ✦ CONNECT...

Many mathematical concepts and skills that your child will learn are based on analyzing and applying patterns. Support your child by using the following Math vocabulary:

- A **pattern** follows a rule, called a **pattern rule**, which tells how the pattern will repeat or change in some way.
- A **number pattern** is an ordered list of numbers that follows a pattern rule. A **shape pattern** is an ordered sequence of shapes that follows a pattern rule.
- Each number or shape in a pattern is called a **term**.
- In a **repeating pattern**, the terms alternate.
  ○ ■ ○ ■ ○ ■
- In a **growing pattern**, each subsequent term increases by the same amount.
  5 10 15 20 25

Activity: Patterns are everywhere in the world around us. Look for patterns in the environment and use them to make a pattern book of number and shape patterns. For each pattern, show several repeating or growing terms. Ask your child to determine the pattern rule, and then follow the rule to extend each pattern using the appropriate terms.

In this unit your child will:

- Interpret multiplication as comparison.
- Use multiplication and division to make comparisons.
- Use more than one step to solve problems.
- Find factors and multiples for whole numbers.
- Make and analyze number and shape patterns.

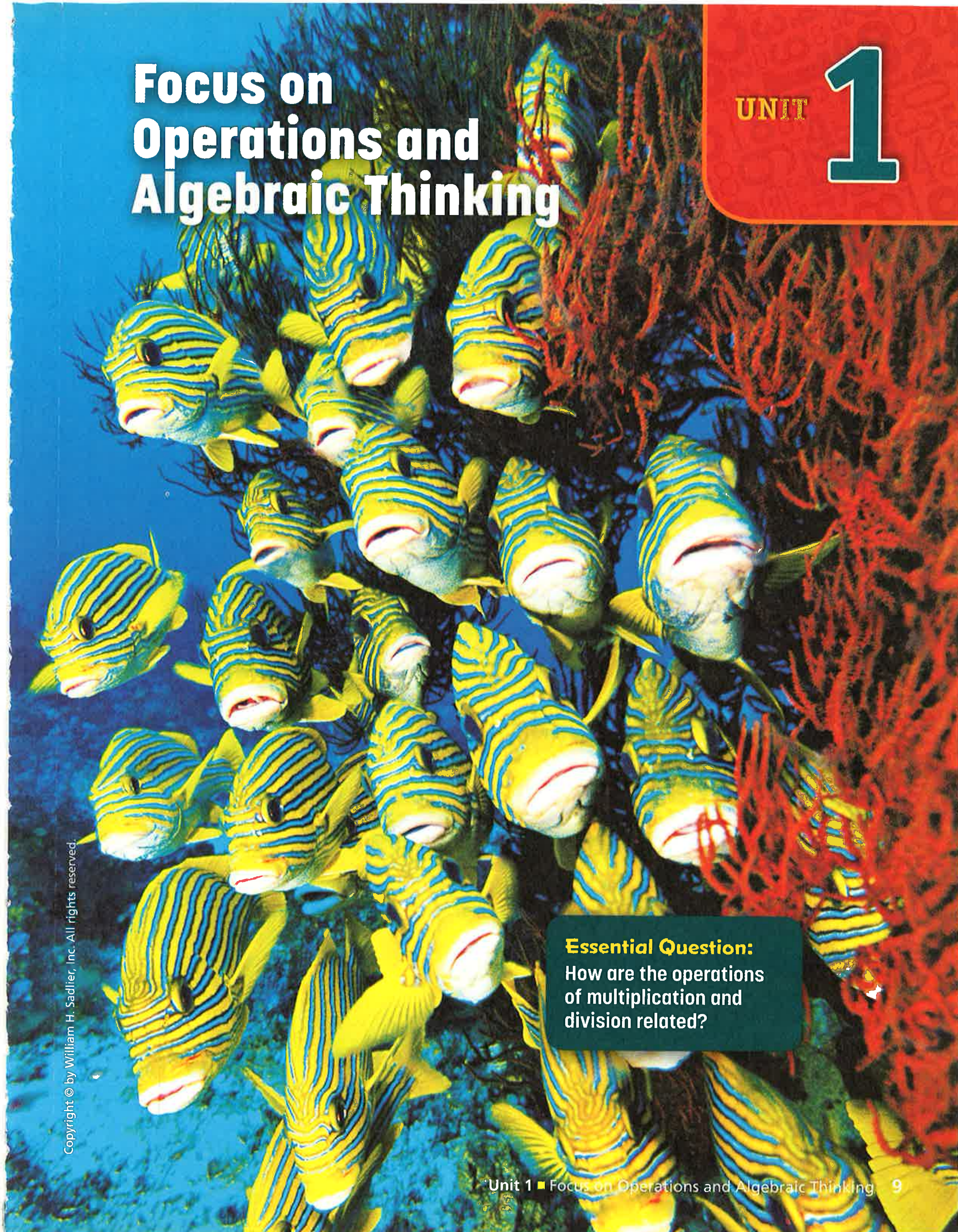
NOTE: All of these learning goals for your child are based on the Grade 4 Common Core State Standards for Mathematics.

Ways to Help Your Child

Help your child strengthen an interest in math by pointing out examples of math in your environment. You can look for patterns, take measurements, count together, and compare prices. A child who appreciates math and learning will look for connections and become a proficient problem solver.

ONLINE

For more Home Connect activities, continue online at
sadlierconnect.com



UNIT 1

Focus on Operations and Algebraic Thinking

Essential Question:
How are the operations of multiplication and division related?

Lesson

1 Interpret Multiplication Equations as Comparisons

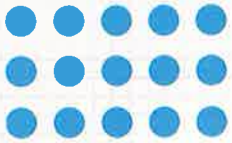
Essential Question:
How can you use multiplication equations to represent comparisons?
4.OA.1

Guided Instruction

In this lesson you will learn about how a multiplication equation can represent comparisons.

Understand: How a multiplication equation represents two comparisons

The array below represents the multiplication equation 15 = 3 × 5.

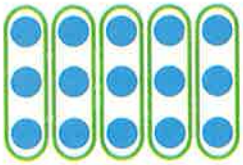


A multiplication
equation shows that:
factor × factor = product
3 × 5 = 15
5 × 3 = 15

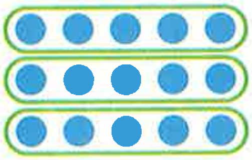
Use the array to help you explain why the equation 15 = 3 × 5 represents these two comparison statements:

"15 is 5 times as many as 3."   "15 is 3 times as many as 5."

There are 3 dots in each column. There are 5 times that many dots in all five columns.
So, the array represents
"15 is 5 times as many as 3."



There are 5 dots in each row. There are 3 times that many dots in all three rows.
So, the array represents
"15 is 3 times as many as 5."



Jamie has 7 times as many pictures as Mary. Mary has 8 pictures. Write a multiplication equation that represents the comparison for this situation.

Guided Instruction

Connect: Multiplication equations and comparisons

Daniel is growing plants in science class. His shortest plant is 4 inches tall. His tallest plant is 20 inches tall.

Write a multiplication equation comparing the heights of Daniel's tallest and shortest plants.

Step 1

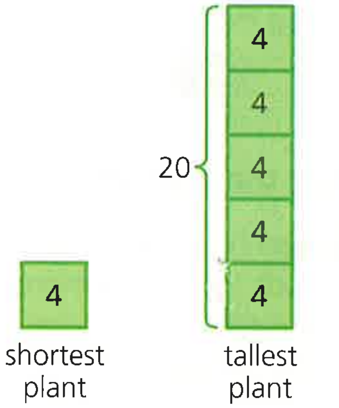
A comparison statement for this situation will be of the form:

height of tallest plant = unknown number × height of shortest plant

We know the heights of the tallest and shortest plants. We do not know the unknown number.

Step 2

Heights of Daniel's Plants



Draw a model and use it to find the unknown information.

First represent 4 inches, the height of the shortest plant. Copy that model until you get 20 inches, the height of the tallest plant. You need 5 copies.

The height of the tallest plant is 5 times the height of the shortest plant.

Step 3

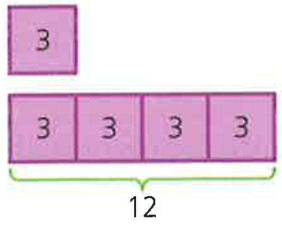
Write a multiplication equation to represent the comparisons.

➡ 20 = 5 × 4 ← 20 inches is 5 times as much as 4 inches.
This equation shows that the height of the tallest plant is 5 times the height of the shortest plant.

✏ What other multiplication equation represents the comparison between Daniel's shortest and tallest plants?

Guided Practice

Complete exercises 1 and 2. Use the model at the right.



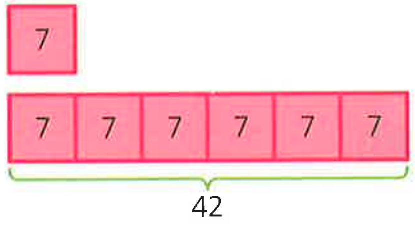
1. Complete the comparison statement to represent the model.

12 is ____ times as many as 3.

2. Complete the multiplication equation to represent the model.

12 = ____ × 3

Complete exercises 3 and 4. Use the model at the right.



3. Complete the comparison statement to represent the model.

____ is ____ times as many as ____.

4. Write a multiplication equation to represent the model.

____

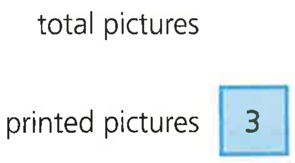
Solve the problem.

5. Lian has a new camera. She printed out 3 pictures for her album. Lian took 9 times as many pictures as she printed out. Write a multiplication equation comparing the number of pictures Lian took to the number she printed out.

____

Complete the model below to justify your answer.

Lian's Pictures

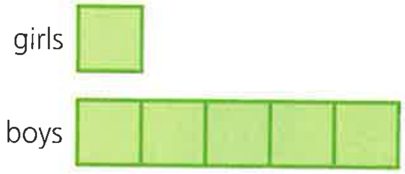


Guided Practice

Use the model. Represent the comparison as a multiplication equation.

6. There are 7 girls and 35 boys on the team. Label the model to show the size of each equal group.

Students on the Team



Count the number of equal groups that make up the total number of boys. Use the information to complete the following sentence.

There are ____ times as many boys as girls.

Compare the numbers of boys and girls. Write a multiplication equation.

_____ = _____ × _____
boys girls

Make a model to help you solve the problem.

7. Katie scored 6 points in the first basketball game of the season. In the second game, she scored 3 times as many points as she scored in the first game. Write a statement in words comparing the numbers of points Katie scored in the two games. Then, write an equation to represent the comparison.

Show your work.

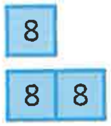
Think • Pair • Share

MP7 **8.** Use a diagram or model to help you explain why 4 times as many as 3 is the same amount as 3 times as many as 4.

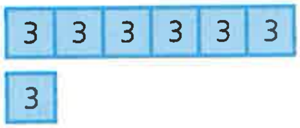
Independent Practice

Write a multiplication equation to represent the comparison shown in the model.

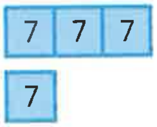
1. ________________



2. ________________



3. ________________



In exercises 4–6, complete the comparison statement that is represented by the equation.

4. $72 = 9 \times 8$

____ is ____ times as many as ____.

5. $49 = 7 \times 7$

____ is ____ times as many as ____.

6. $36 = 4 \times 9$

____ is ____ times as many as ____.

In exercises 7 and 8, solve the problem.

7. A sweater costs $40. This is 4 times the amount of money that Beth has. Write a multiplication equation to represent the comparison between the price of the sweater and the amount Beth has.

Answer ________________________________

Draw a model to justify your answer.

8. Ramon has read 6 pages in his book. He needs to read 8 times as many pages to finish the book. Write a multiplication equation to represent the comparison between the number of pages Ramon has read and the number he has to read to finish the book.

Answer ________________________________

Draw a model to justify your answer.

Independent Practice

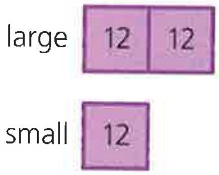
For exercises 9 and 10, use the model at the right to write a multiplication equation.

9. A large paint set has 2 times as many colors as a small set. A small paint set has 12 colors.

Write a multiplication equation for the comparison.

24 = ________

Paint Sets



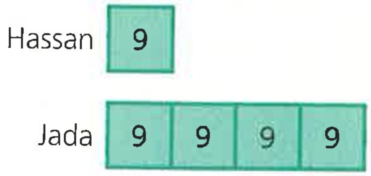
10. Hassan lives 9 blocks from school. Jada lives 36 blocks from school. Complete the following sentence.

Jada lives ____ times as far from school than Hassan does.

Write a multiplication equation for the comparison.

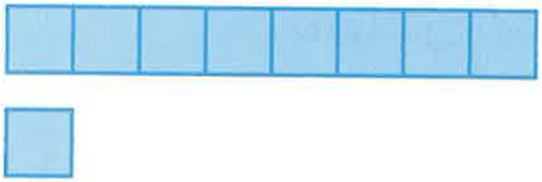
____ × ____ = ____

Blocks from School



11. The tree in Derrick's front yard is 8 times as old as the tree in his backyard. The tree in the front yard is 72 years old. Label the model to show the age of each tree.

Derrick's Trees



For exercises 12–14, use the model above.

12. How old is the tree in Derrick's backyard?

Answer ________________________________

13. How old is the tree in Derrick's front yard?

a. 8 years

b. 9 years

c. 10 years

d. 72 years

14. Which multiplication equation correctly compares the ages of the two trees?

a. 10 × 8 = 80

b. 8 × 9 = 72

c. 6 × 12 = 72

d. 9 × 9 = 81

Independent Practice

MP4 **15.** Give an example of a situation in which you would need to compare two amounts using multiplication. What multiplication equation shows the relationship between the numbers in your example?

MP7 **16.** In a multiplication equation, is it always important to know which amount represents the number of groups and which represents the size of each group? Give one example.

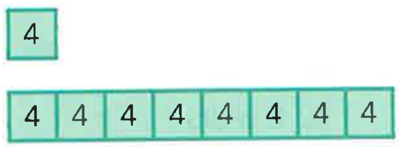
Solve the problems.

MP4 **17.** Last year, Ben's family went on vacation for 12 days. This year, the family is traveling for 5 times as many days as last year. Use your own method to find how long the family's trip will be.

Show your work.

Answer ______________________________

MP3 **18.** Nina draws the model at the right to represent the relationship 8 times as many as 4 is 32. Nina thinks she can label the model to represent 8 × 2 = 16. Is Nina correct?



Show your work.

Answer ______________________________

Independent Practice

Solve the problems.

MP6 **19.** One plant is 12 inches tall and another is 3 inches tall. Sam says "The height of the tall plant is 4 times as much as the height of the short plant." Meg says "The height of the tall plant is 9 inches more than the height of the small plant." Who is correct?

Answer ______________________

Justify your answer using words, drawings, or numbers.

MP5 **20.** Huey walks neighborhood dogs to earn extra money. He walks 2 dogs each week. Huey says that he can walk 4 times *more* dogs each week. Is this the same as saying that Huey can walk 4 times *as many* dogs each week?

Answer ______________________

Justify your answer using words, drawings, or numbers.

MP4 **21.** A summer camp has 5 cabins. There are 10 single beds in each cabin. So there are 10 times as many beds as cabins. If 55 campers sign up for camp, how many more beds will be needed?

Show your work.

Answer ______________________

Lesson 2 Problem Solving: Use Multiplication and Division to Make Comparisons

Essential Question:
How can you solve problems involving multiplicative comparisons?
4.OA.2

Guided Instruction

In this lesson you will learn how to solve problems involving multiplicative comparison.

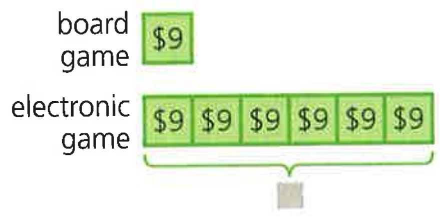
Understand: Comparison with an unknown product

A board game costs $9. An electronic game costs 6 times as much as the board game. How much does the electronic game cost?

You can start by making a model of the problem, like the one shown at the right. Use a ■ for the unknown amount.

Next, write and solve a multiplication equation to find the cost of the electronic game.

Game Prices



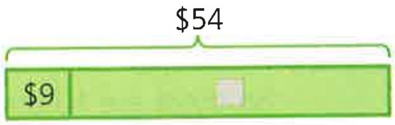
6 times as much as $9 = cost of electronic game
6 × $9 = ■ ← Multiply to find the unknown product.
6 × $9 = $54

➡ The electronic game costs $54.

Understand: Comparisons can involve addition

A board game costs $9. An electronic game costs $54. How much more does the electronic game cost than the board game?

A model of this problem is shown at the right.



Write and solve an equation.

$9 + ■ = $54 ← $9 *plus* unknown amount equals $54.
$54 − $9 = ■ ← Write an equivalent subtraction equation.
$54 − $9 = $45

➡ The electronic game cost $45 more than the board game.

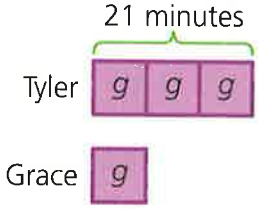
✏ Compare these statements: $54 is *6 times as much as* $9.
$54 is *$45 more than* $9.

Guided Instruction

Understand: Comparison with an unknown factor

Tyler waits 21 minutes for the school bus. Tyler's waiting time is 3 times as long as Grace's waiting time. How long does Grace wait for the bus?

To find how long Grace waits for the bus, first model the problem. Then write and solve an equation. Use g to represent the unknown quantity, Grace's waiting time.


Waiting Times

Grace's waiting time is the amount being multiplied, or the size of each group in the model.

Write a multiplication equation for the comparison.

21 = 3 times as long as Grace's waiting time

$21 = 3 \times g$

Write and solve a related division equation to find g, the unknown factor.

$21 \div 3 = g$

$7 = g$

Remember!
Multiplication and division are opposite, or *inverse*, operations.

Grace waits 7 minutes for the bus.

Find how many minutes more Tyler waited than Grace waited.

Lesson 2 Problem Solving: Use Multiplication and Division to Make Comparisons

Guided Instruction

Connect: What you know about comparison and multiplication

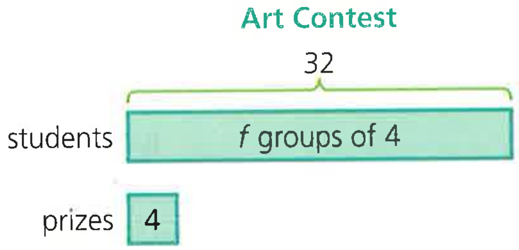
There are 32 students in an art contest. Only 4 prizes will be given. How many times the number of prizes is the number of students?

To solve, make a model and write an equation.

Step 1

Make a model to help you understand the situation.

You need to find the factor that you must multiply 4 by to get 32.



Step 2

Write an equation. Use f to represent the unknown factor.

32 is an unknown factor times as many as 4
$32 = f \times 4$

Step 3

Write a related division equation to find f.

$32 \div 4 = f$
$8 = f$

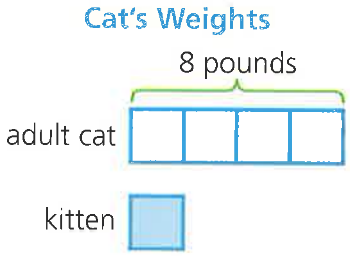
Remember!
factor × factor = product
product ÷ factor = factor

There are 8 times as many students as prizes.

In another contest, there are 42 students. There are 7 times as many students as prizes. How many prizes are there?

Guided Practice

Use the model at the right to solve the problem.


Cat's Weights

1. An adult cat weighs 8 pounds, and is 4 times heavier than a kitten. How much does the kitten weigh?

a. Identify the numbers given in the problem.

the greater amount, or product: ____ pounds

the factor telling the number of groups in 8: ____

b. What do you need to find? Describe the unknown factor.

____

c. Write a letter to represent the unknown factor.

____

Use this same letter for exercises d and e below.

d. Write a multiplication equation for the comparison.

8 = ____ × ____

e. Write a related division equation to find the unknown factor.

____ ÷ ____ = ____

f. Use your division equation to solve, and label the model above with your answer. Then complete the following sentence.

The kitten weighs ____ pounds.

g. How do you know that your answer is correct?

____

Think • Pair • Share

MP3 **2.** Why is it helpful to represent a multiplicative comparison problem with both a multiplication equation and a division equation? Explain.

Independent Practice

Write an equation you can use to solve for u, the unknown number. Then solve.

1. $2 \times u = 8$

____ ÷ ____ $= u$

____ $= u$

2. $u \times 7 = 49$

__________ $= u$

____ $= u$

3. $9 \times 10 = u$

__________ $= u$

____ $= u$

Describe the unknown number, y, in each problem. Then write the letter of the matching equation.

a. $6 \div 3 = y$

b. $6 - 3 = y$

c. $6 \times 3 = y$

d. $6 + 3 = y$

4. A fruit salad has 6 strawberries and 3 times as many blueberries. How many blueberries are there?

y is ______________________________

Matching equation: ____

5. Diego orders 3 books online. The store mails him 6 books instead. How many more books does Diego have than he ordered?

y is ______________________________

Matching equation: ____

6. A rectangular sign is 6 feet long. This is 3 times as long as the width of the sign. What is the width of the sign?

y is ______________________________

Matching equation: ____

7. Emma's soccer team scores 3 goals. The winning team scores 6 goals. How many times as many goals does the winning team score as Emma's team?

y is ______________________________

Matching equation: ____

Independent Practice

Write an equation with a letter for the unknown number. Then solve the problem. You can use the model to help.

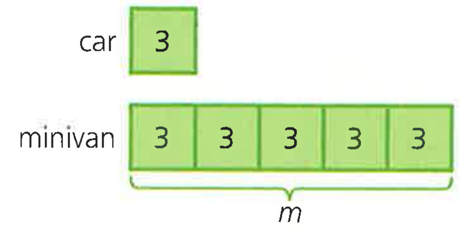
8. Caleb's family has a car that is 3 years old. They also have a minivan that is 5 times as old as the car. How old is the minivan?

Equation ______________________

Answer ______________________

____

Automobile Ages



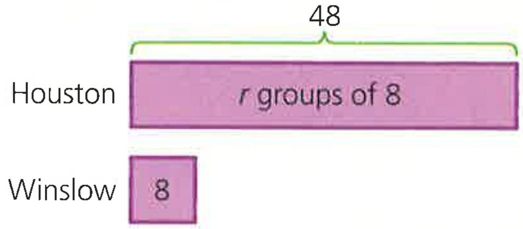
9. Houston gets about 48 inches of rain a year. Winslow gets about 8 inches of rain a year. How many times as much rain does Houston get as Winslow?

Equation ______________________

Answer ______________________

____

Annual Rainfall in Inches



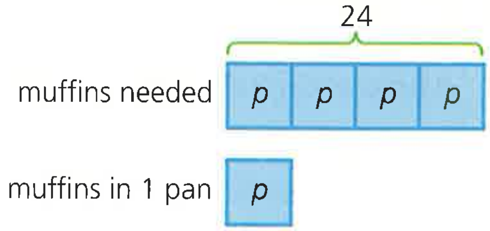
10. Aliyah needs to bake 24 muffins. This is 4 times as many muffins as her muffin pan holds. How many muffins does Aliyah's pan hold?

Equation ______________________

Answer ______________________

____

Aliyah's Muffins



Circle the correct answer.

11. Jin has a muffin pan that holds 12 muffins. He lends the pan to Aliyah, who needs to bake 24 muffins. How many times as many muffins are needed as Jin's pan can hold?

a. 2 times as many

b. 3 times as many

c. 4 times as many

d. 6 times as many

Independent Practice

Draw a model and write an equation to represent each problem. Then solve.

MP1 **12.** Dmitri has saved $9 to buy a skateboard. The one he wants costs 8 times as much as his savings. How much does the skateboard cost?

Show your work.

Answer ______________________

MP2 **13.** An office building has 40 floors. It is 5 times as tall as the apartment building across the street. How many floors does the apartment building have?

Show your work.

Answer ______________________

Answer each question in a complete sentence.

MP6 **14.** The deep end of a pool is 12 feet deep. The shallow end is 3 feet deep. Noah solved the equation $12 \div 3 = p$ to find that the deep end is 4 times as deep as the shallow end. Explain how Noah can check his answer.

MP3 **15.** Serena says that all multiplicative comparison problems are solved by multiplying the given numbers. Do you agree? Explain your answer.

Independent Practice

Solve the problems.

MP4 **16.** Eli collects stamps. He has 63 stamps from the United States. He also has 7 stamps from Canada. How many times as many U.S. stamps does Eli have as Canadian stamps?

Show your work.

Answer ______________________________

MP1 **17.** Olivia says that she used the computer for 9 hours this week. She thinks her brother Jack used it for 3 times as long as she did. Jack says he used it for 12 more hours than Olivia. Who counts a greater number of hours for Jack?

Answer ______________________________

Justify your answer using words, drawings, or numbers.

MP4 **18.** Olivia and Jack's father says that they used the computer for 30 hours altogether. He tells them that Jack used the computer for 2 times as long as Olivia. If this is true, did Olivia use the computer for 9 hours?

Answer ______________________________

Justify your answer using words, drawings, or numbers.

Lesson

3 Problem Solving: Multistep Problems

Essential Question:
How can equations help you solve multistep problems?
4.OA.3

Words to Know:
remainder

Guided Instruction

In this lesson you will learn how to use equations to solve problems with more than one step.

Understand: Equations to solve multistep problems

The fourth graders are having a picnic. Two classes of 30 students each, 2 teachers, and 2 parents will be there. If 8 people sit at each table, how many tables are needed for all the people at the picnic?

To find the number of tables needed, you divide the total number of people by the number of people who can sit at each table.
To solve this problem, first write a statement or a word equation.

$$\text{number of tables needed} = \text{total number of people} \div \text{number who can sit at each table}$$

Next, think about the three quantities in the equation:
The number of tables needed is unknown. You can use *t* to represent this number. The total number of people is the number of students in the two classes plus the number of adults: $2 \times 30 + 2 + 2$
The number of people at each table is 8.

Use this information to rewrite an equation.

$$t = \underbrace{(2 \times 30 + 2 + 2)}_{\text{2 groups of 30 students plus 2 teachers and 2 parents}} \div 8 \longleftarrow$$

Use parentheses to show that the total number of people is calculated *before* dividing.

Now, solve the equation. Use the order of operations.

$t = (2 \times 30 + 2 + 2) \div 8$
$t = (60 + 2 + 2) \div 8$ Multiply within the parentheses.
$t = 64 \div 8$ Add within the parentheses.
$t = 8$ Divide.

Check that your answer is reasonable. You can use reasoning and mental math. Eight tables can hold 8 × 8, or 64 people. There will be 4 adults plus 2 × 30, or 60 students, which is a total of 64 people. The answer is correct.

➡ Eight picnic tables are needed for all the people at the picnic.

Understand: The meaning of a remainder

> Amanda collects old postcards. She had 71 postcards. Then, she sold 19 of them to another collector. She wants to put the postcards she has left in an album that fits 6 cards on each page. How many pages will Amanda need to hold the postcards she has left?

To find the number of pages Amanda needs, write a word equation for the problem.

number of pages = number of postcards ÷ number that fit on each page

You can use p to represent the unknown number of pages. Use the information from the problem to write the equation.

$$p = \underbrace{(71 - 19)}_{\text{71 cards she started with minus the 19 she sold}} \div 6 \longleftarrow \text{Use parentheses to show that the subtraction is done \textit{before} the division.}$$

Now, solve the equation.

$p = (71 - 19) \div 6$

$p = 52 \div 6$ Subtract within the parentheses.

$p = 52 \div 6 = 8\text{ R}4$ Divide. Read as, "52 divided by 6 equals 8 remainder 4."

There are 8 groups of 6 in 52 with 4 left over. The amount left over after dividing is called the remainder. The remainder is written with an R before it.

Amanda can fill 8 whole pages. She will need 1 more page for the remaining 4 postcards. So, she needs 9 pages in all.

➡ Amanda will need 9 pages to hold the postcards she has left.

Guided Instruction

Connect: Problem situations and reasonable solutions

On the first night of the school play, 371 tickets were sold. On the second night, 59 fewer tickets were sold than the first night. How many tickets were sold altogether for both nights of the play?

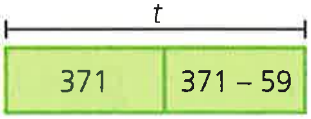
To find how many tickets were sold, write and solve an equation.

Step 1

Write a word equation to represent the problem.
total tickets sold = tickets sold the first night + tickets sold the second night

Step 2

Use the information from the problem to write an equation. You can use a model to help you understand how the quantities are related. Notice that you need to subtract to find the number of tickets sold on the second night.



$$t = 371 + \underbrace{(371 - 59)}_{\text{59 fewer tickets than on the first night}}$$

Step 3

Solve the equation.

$t = 371 + (371 - 59)$
$t = 371 + 312$ Subtract within parentheses.
$t = 683$ Add.

Step 4

Use rounding and estimation to check that the answer is reasonable. 371 is about 370 and 59 is about 60, so the answer should be about:
$370 + (370 - 60) = 370 + 310 = 680$

Remember!
An estimate is a number that is close to an exact amount.

The estimate 680 is close to the actual answer 683. So the answer is reasonable.

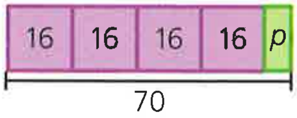
➡ A total of 683 tickets were sold altogether for both nights of the play.

Guided Practice

For exercises 1–6, follow the steps to solve the problem. Use the model if it helps you.

There are 70 people waiting to tour a museum. The tour guide takes 4 groups with 16 people each. The remaining people will be in the last tour group. How many people are in the last tour group?

Museum Tour Groups



1. Use the following terms to make a word equation for this problem: *people in first 4 groups*, *people waiting*, *people in last group*.

   ________ = ________ − ________

2. There are ____ people waiting.

3. Show the number of people in the first four groups.

   ____ × 16

4. Use p to represent the number of people in the last group. Then write an equation to find how many people are in the last group using the information from the problem.

   $p =$ ____ − (________)

5. Solve the equation.

   $p =$ ____ − (________)

   $p =$ ____ − ____

   $p =$ ____

6. There are ____ people in the last tour group.

Think • Pair • Share

MP2 7. Molly says that the answer for the problem above is the same as the remainder for 70 ÷ 16. Is Molly correct? You can use the model to help you explain your answer.

Independent Practice

Use the steps to model and solve each problem. Then estimate to check your answer.

1. Three hikers want to share the weight in their backpacks equally. Their backpacks weigh 32 pounds, 39 pounds, and 22 pounds. How much weight should each hiker carry?

   Write a word equation.

   weight each person carries = total weight ÷ ____________

   Use *b* for the unknown. Use the information from the problem to rewrite the equation.

   $b = (\_\_\_ + \_\_\_ + \_\_\_) \div \_\_\_$

   Solve the equation to find the answer.

   Answer ____________________

   Use estimation to check your answer.

2. Jacob has read 39 library books since school began. He plans to read another 2 books a week for the next 12 weeks. At the end of this time, how many books will he have read altogether?

   Write a word equation.

   ____________ = books read so far + ____________

   Use *t* for the unknown. Substitute the information from the problem.

   $t = \_\_\_ + \_\_\_\_\_\_$

   Solve the equation to find the answer.

   Answer ____________________

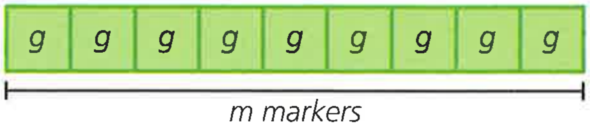
   Use estimation to check your answer.

Independent Practice

In exercises 3–5, follow the steps to solve the problem. Use the model at the right if it helps you.

Tina has 6 packs of 8 colored markers to put into party gift bags. She splits the markers equally among 9 gift bags. How many markers can she put in each bag?

Party Gift Bags



3. Write an equation for this problem.

$g =$ ______________

4. Solve the equation.

5. Answer the question.

Tina can put _____ markers in each bag.

6. Is the final answer in exercise 5 the same as the solution to your equation? Explain why or why not.

Circle the correct answer.

7. Peter practices the piano for 25 minutes 3 times a week. He also has a 55-minute piano lesson every Saturday. How much time does Peter spend playing the piano in one week?

a. 80 minutes
b. 130 minutes
c. 190 minutes
d. 240 minutes

8. Sixteen teams of 5 students each sign up for a sports tournament. The principal will combine the teams and then divide the students into 10 new teams. How many students will be on each of the new teams?

a. 7
b. 7 R5
c. 8
d. 10

Independent Practice

MP6 **9.** Haj solved the equation $w = 718 - (329 + 178)$. She got the solution $w = 567$. Use estimation to check Haj's solution. Do you think her solution is reasonable? Explain.

MP1 **10.** Consider this division problem: $27 \div 4 = 6$ R3. Describe a real-world problem that can be solved by finding $27 \div 4$. Give the solution to your problem. How did you think about the remainder when you found the solution?

Solve the problem.

MP4 **11.** A total of 96 students vote on two choices for a field trip. The zoo gets 2 times as many votes as the radio station. How many students vote to go to the zoo?

Show your work.

Answer ______________________________

Independent Practice

Solve the problems.

MP4 **12.** A soccer team orders 36 new jerseys for games. Half of the jerseys are red and the other half are white. Five of the white jerseys the team receives are the wrong size. How many white jerseys are the correct size?

Show your work.

Answer ____________________

MP7 **13.** There are 29, 15, 31, and 25 people in four groups of volunteers. Colin wants to divide the volunteers into 5 equal teams to clean up different areas of the park. The following equation can be used to find the number of people on each team:

$p = (29 + 15 + 31 + 25) \div 5$

Rewrite this equation so it is easy to solve mentally. Then solve the problem in your head. Explain what you did.

Answer ____________________

Justify your answer using words, drawings, or numbers.

MP6 **14.** A water faucet that drips 10 times a minute wastes 3 liters of water in 1 day. If 2 of the faucets in Charlotte's house drip at this rate, how many days will it take to waste 100 liters of water?

Answer ____________________

Justify your answer using words, drawings, or numbers.

Lesson 4

Find Factors and Multiples for Whole Numbers

Essential Question:
What are factors and multiples of whole numbers?
4.OA.4

Words to Know:
- factor
- factor pair
- prime number
- composite number
- multiple

Guided Instruction

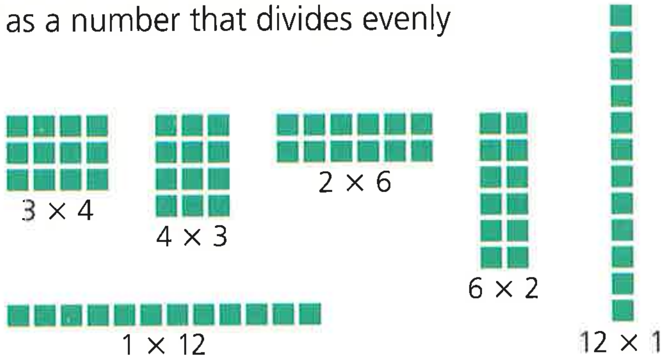
Understand: Factors and factor pairs

Mr. Yoo has 12 student worksheets to display. He wants to display them in rows. He wants the same number of worksheets in each row. How many different ways can Mr. Yoo display the worksheets?

Model all the ways that the 12 worksheets can be displayed in a rectangular array. To do this, find the pairs of whole numbers with a product of 12.

Factors are numbers that are multiplied together to make a product. A factor of a number can also be thought of as a number that divides evenly into that number, without a remainder.
The whole number factors of 12 are 1, 2, 3, 4, 6, and 12.

The factor pairs for 12 are: 1 and 12, 2 and 6, 3 and 4.
You can make two arrays for each factor pair.



➡ Mr. Yoo can display the worksheets in 6 different ways.

Understand: Prime and Composite Numbers

Suppose Mr. Yoo has 13 pieces of student worksheets. In how many ways can Mr. Yoo display the worksheets in equal rows?

The only factor pair for 13 is 1 and 13. So Mr. Yoo can display the worksheets as a 1 × 13 array or as a 13 × 1 array.

The number 13 is an example of a prime number. A prime number is a number greater than 1 with only two factors, 1 and the number itself. The number 12 is a composite number. A composite number is a number greater than 1 that has more than two factors.

➡ Mr. Yoo can display the worksheets in 2 different ways.

Guided Instruction

Understand: Finding factors of a whole number

Find all the factors of 15. Tell whether 15 is prime or composite.

Remember!
A factor of a whole number divides evenly into the number.

To find the factors of 15, divide 15 by whole numbers, starting with 1. If a number divides into 15 evenly, then both it and the quotient are factors of 15. Keep checking until the factors start to repeat.

Is 15 divisible by 1?	by 2?	by 3?	by 4?
Yes, 15 ÷ 1 = 15 1 and 15 are factors.	No	Yes, 15 ÷ 3 = 5 3 and 5 are factors.	No

The next number to check is 5, but you already found that 5 is a factor of 15. You can stop checking. The factors of 15 are 1, 3, 5, and 15.

The factors of 15 are 1, 3, 5, and 15. It is a composite number.

Understand: Finding multiples of a whole number

Students sign up for a competition in teams of 3. What are some possibilities for the total number of students in the competition?

1 team is 1 × 3 = 3 students
2 teams is 2 × 3 = 6 students
3 teams is 3 × 3 = 9 students
4 teams is 4 × 3 = 12 students
5 teams is 5 × 3 = 15 students
6 teams is 6 × 3 = 18 students

The numbers 3, 6, 9, 12, 15, and 18 are multiples of 3. A multiple of a whole number is the product of that number and another whole number. Notice that 3 is a factor of each of its multiples.

The possible numbers of students are the multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, and so on.

Find all the factors of 19 and 24. Tell whether each is prime or composite.

Guided Instruction

Connect: Factors and multiples

Juice boxes are sold in packs of 8. Snack bags of crackers are sold in packs of 6. Ms. Leone takes one of each in her lunch. What is the least number of packs of each Ms. Leone can buy to have the same number of juice boxes and bags of crackers?

To solve use multiples of 8 and 6.

Step 1

Use multiples to find the possible numbers of each item Ms. Leone can buy.

There are 8 juice boxes in each pack, so Ms. Leone can only buy juice boxes in multiples of 8. List the first several multiples of 8:

$$\begin{array}{cccccccc} 8 & 16 & 24 & 32 & 40 & 48 & 56 & 64 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 1 \times 8 & 2 \times 8 & 3 \times 8 & 4 \times 8 & 5 \times 8 & 6 \times 8 & 7 \times 8 & 8 \times 8 \end{array}$$

There are 6 bags of crackers in every pack, so Ms. Leone can only buy bags of crackers in multiplies of 6. List the first several multiples of 6:

$$\begin{array}{cccccccc} 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 1 \times 6 & 2 \times 6 & 3 \times 6 & 4 \times 6 & 5 \times 6 & 6 \times 6 & 7 \times 6 & 8 \times 6 \end{array}$$

Step 2

Find the least number that is both a multiple of 8 and a multiple of 6.

The least number that is both a multiple of 8 and a multiple of 6 is 24. Ms. Leone can buy exactly 24 boxes of juice and 24 bags of crackers.

$3 \times 8 = 24$, she must buy 3 packs of juice boxes to get 24 boxes.
$4 \times 6 = 24$, she must buy 4 packs of crackers to get 24 bags.

Notice the connection between factors and multiples:
24 is a multiple of 8 and 8 is a factor of 24.
Similarly, 24 is a multiple of 6 and 6 is a factor of 24.

In general, a whole number is a multiple of each of its factors.

➡ The least number of packs Ms. Leone can buy is 3 packs of juice boxes and 4 packs of crackers.

Guided Practice

Make all the possible arrays for the number. Then, list the factors and circle *prime* or *composite*.

1. 10

Arrays:

Factors: ____________________

prime or ***composite***

2. Complete this table to find the factors of 35. Use the first two rows as a guide. Some cells in the last column will be empty.

Is 35 divisible by 1?	Yes, 35 ÷ 1 = 35	1 and 35 are factors.
Is 35 divisible by 2?	No	
Is 35 divisible by 3?		
Is 35 divisible by 4?		
Is 35 divisible by 5?		
Is 35 divisible by 6?		

Why don't you need to check 7? ____________________

List all the factors. ____________________

Think • Pair • Share

MP2 3. Cole said, "All prime numbers are odd and all composite numbers are even." Do you agree with Cole? Explain.

Independent Practice

Make all the possible arrays for the number. Then, list the factors and circle *prime* or *composite*.

1. 11

Arrays:

Factors: ____________________

prime or ***composite***

2. 16

Arrays:

Factors: ____________________

prime or ***composite***

3. Complete this table to find the factors of 18. Use the first two rows as a guide. Some cells in the last column will be empty.

Is 18 divisible by 1?	Yes, 18 ÷ 1 = 18	____ and ____ are factors.
Is 18 divisible by 2?		
Is 18 divisible by 3?		
Is 18 divisible by 4?		
Is 18 divisible by 5?		

Why don't you need to check 6? ____________________

List all the factors. ____________________

Independent Practice

For exercises 4–7, find all the factors of the number. Show your work. Then circle *prime* or *composite*.

4. 22

Factors: ____________________

prime or ***composite***

5. 23

Factors: ____________________

prime or ***composite***

6. 36

Factors: ____________________

prime or ***composite***

7. 50

Factors: ____________________

prime or ***composite***

For exercises 8 and 9, list the first six multiples of the number.

8. 7

Multiples: ____________________

9. 5

Multiples: ____________________

For exercises 10 and 11, circle the correct answer or answers.

10. Which of these numbers are multiples of 9?

a. 9
b. 38
c. 81
d. 84

11. Which of these numbers are prime numbers?

a. 2
b. 19
c. 52
d. 77

12. Mandy says that the factors of 20 are 2, 4, 5, and 10. What mistake did Mandy make?

____

Independent Practice

MP7 **13.** Lily learns that all numbers that have 0 or 5 as the last digit have 5 as a factor. Explain why this is true.

MP7 **14.** David says that finding all the numbers that 28 is a multiple of is the same as finding the factors of 28. Is David correct? Explain.

Solve the problems.

MP7 **15.** Amelia says that 50 must have more factors than 40, since it is a greater number. Is Amelia correct? Find the factors of 50 and 40.

Show your work.

Answer ______________________________

MP7 **16.** A multiplication table shows that 42 is a multiple of both 6 and 7. What other numbers have 42 as a multiple?

Show your work.

Answer ______________________________

Independent Practice

Solve the problems.

MP6 **17.** A farmer has 96 apples to sell at the farmers market. She wants to bag the apples before going. How many ways can the farmer bag the apples so that:

- each bag has at least 4 apples
- each bag has no more than 10 apples
- all bags have the same number of apples?

Show your work.

Answer ____________________

MP4 **18.** A chef makes 88 ounces of jam. He wants to put the jam in 6-ounce jars. Can he use this size jar if each must be completely filled with jam, without any jam left over? If not, tell what sizes the chef can use.

Answer ____________________

Justify your answer using words, drawings, or numbers.

MP1 **19.** Andy set his computer to beep every 6 minutes and to chirp every 10 minutes. His computer just beeped and chirped at the same time. In how many minutes will it next make both sounds at the same time?

Answer ____________________

Justify your answer using words, drawings, or numbers.

Lesson 5

Generate and Analyze Number and Shape Patterns

Essential Question:
How can you create and describe patterns?
4.OA.5

Words to Know:
number pattern
pattern rule
term
shape pattern

Guided Instruction

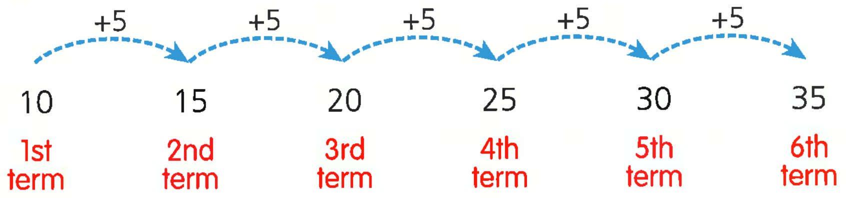
In this lesson you will learn how to make, analyze, and describe number and shape patterns.

Understand: Number patterns and pattern rules

Maria is saving money for a new violin. She starts with \$10 and saves \$5 the first week and each week thereafter. How much money does Maria have each week for the first five weeks?

To find the money amounts, you can make a number pattern. A number pattern is an ordered list of numbers that follow a rule and repeat or change in some way. The pattern rule tells you how the pattern works.

To generate the pattern for Maria's savings, start at 10 and follow the rule *add* 5. Each number in a number pattern is called a term. This pattern tells you that each new term *increases* by 5, so this is a growing pattern.



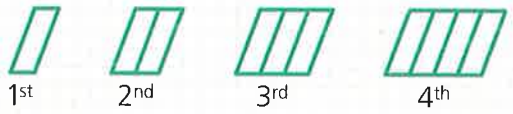
➡ For the first five weeks, Maria will have \$15, \$20, \$25, \$30, and \$35.

✎ Use the same rule to make a new number pattern that starts with 1 instead of 10. How does the pattern compare to Maria's pattern?

Guided Instruction

Understand: Growing shape patterns

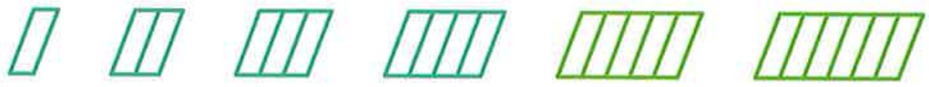
Maria uses parallelograms to make a pattern of growing shapes.



What is the rule for Maria's pattern? What are the next two terms?

A shape pattern is an ordered sequence of shapes that follow a rule. Each shape in a shape pattern is called a term. Maria's shape pattern starts with one parallelogram and follows the rule *attach 1 parallelogram to the right side of the previous term*.

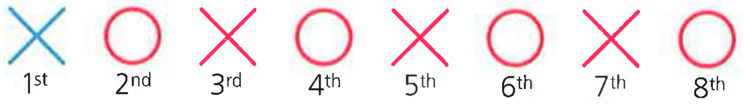
Here is Maria's pattern with two more terms.



Understand: Repeating shape patterns

Start with the first term below and use the following rule to make a pattern: *If the term before is an X, make an O. If the term before is an O, make an X.*

Start with X and follow the rule. The pattern goes back and forth, or alternates, between X and O. Here are the first eight terms.



In this pattern all the odd-numbered terms are X-shapes. All the even-numbered terms are circles.

Guided Instruction

Connect: Making patterns and analyzing terms

Maria makes a new number pattern. She starts with 2 and uses the rule *multiply by 2*. What are the first six terms for Maria's pattern? Look for other patterns and relationships in the terms and record your observations.

To find the first six terms generate the pattern.

➡ To generate the first six terms, start with 2 and multiply the previous term by 2 to get the next term.

2, 4, 8, 16, 32, 64, . . .

Observation 1: The terms are all even numbers and increase by doubling.

Look at how the terms increase. Subtract each term from the next term to find their difference.

terms	2		4		8		16		32		64
difference		2		4		8		16		32	

Observation 2: The difference between each two terms is the same as the lesser term. The same pattern is formed by the differences.

Look for a relationship between each term and its order in the pattern.

	Terms	
1st term	2	= 2
2nd term	4	= 2 × 2
3rd term	8	= 2 × 2 × 2
4th term	16	= 2 × 2 × 2 × 2
5th term	32	= 2 × 2 × 2 × 2 × 2
6th term	64	= 2 × 2 × 2 × 2 × 2 × 2

Observation 3: Each term's place in the pattern tells you the number of times 2 is multiplied by itself.

Guided Practice

Use the rule to write or draw the missing terms in each pattern. Then complete the sentence below each exercise.

1. Start with 1 and follow the rule *add 2*.

1, ____, 5, ____, 9, ____, . . .

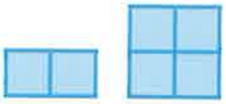
All of the terms are ____________.
even or odd?

2. Start with 50 and follow the rule *subtract 5*.

____, 45, ____, 35, ____, . . .

All the terms are multiples of ____.

3. Start with a row of 2 unit squares and follow the rule *add a row of 2 unit squares on top of the shape*.



This is a ________________ shape pattern. The height of the shape
growing or repeating

increases by ____ unit with each term.

Think • Pair • Share

MP7 **4.** Create a new pattern by adding the corresponding terms in the patterns from exercises 1 and 2. That is, add 1 and 50 to get the first term, add 3 and 45 to get the second term, and so on. The new pattern starts with 51. Show the pattern. What is the rule for the pattern? Explain why this rule makes sense?

Independent Practice

Identify the rule for the pattern. Write the letter of the correct rule.

1. 1, 5, 9, 13, 17, . . . ____	**a.** Multiply by 4
	b. Add 2
2. 3, 6, 12, 24, 48, 96, . . . ____	**c.** Add 4
3. 1, 4, 16, 64, 256, . . . ____	**d.** Add 3
	e. Multiply by 2
4. 4, 7, 10, 13, 16, . . . ____	

Use the rule to write or draw the missing terms in each pattern.

5. Start with 10 and add 5.

10, ____, 20, ____, 30, ____, . . .

6. Add the digits in each term in exercise 5 to make a new pattern.

1, ____, ____, ____, ____, ____, . . .

7. Start with 1 and multiply by 3.

1, ____, ____, ____, 81, . . .

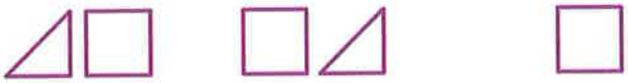
8. Make a new pattern by finding the differences of consecutive terms for the pattern in exercise 7.

2, ____, ____, ____, . . .

9. What is the rule for the new pattern in exercise 8?

____

10. Start with a triangle and follow this rule: *If the term before is a triangle, make a square. If the term before is a square, make a triangle.*



11. What pattern is shown by the number of sides in the shapes in exercise 10?

____

12. The pattern in exercises 10 and 11 are ______________.
growing or repeating

Independent Practice

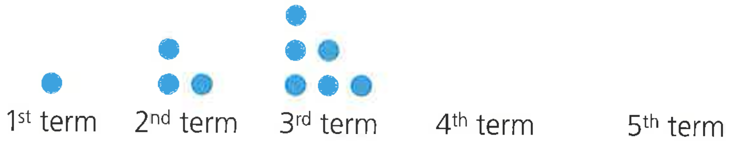
For exercises 13–15, use the rule to generate the pattern. Then write at least two observations about the pattern.

MP7 **13.** Start with 6 and use the rule *add 3*.

6, ____, ____, ____, ____, ____, . . .

Observations

MP2 **14.** Continue the growing pattern of dots. How many dots will be in the bottom row of the 8th term?



Answer ________________________________

Observations

MP6 **15.** Start with 3. Use the rule *multiply by 2 and then subtract 1*.

3, 5, ____, ____, ____, ____, . . .

Observations

MP4 **16.** Marley generated this pattern using an addition rule. One term in the pattern is incorrect.

7, 15, 25, 31, 39, 47, . . .

Which is the incorrect term?

a. 7 **b.** 25 **c.** 39 **d.** 47

Justify your answer using words, drawings, or numbers.

Independent Practice

Solve the problems.

MP3 **17.** Tanya said, "If two patterns are generated using the same rule, then they must be the same." Is Tanya correct? Explain.

MP3 **18.** Michael says that number patterns cannot use subtraction. Do you agree? Give a simple example in your answer.

MP7 **19.** Haley is studying for a spelling bee. Her goal is to learn twice as many words each day as on the previous day. To show her study schedule for the next six days, she makes this number pattern for the number of words.

10, 20, 30, 40, 50, 60

If Haley learns this many words each day, will she meet her goal?

Show your work.

Answer ______________________________

MP8 **20.** Justin's neighbor hired him to mow her lawn each week for 6 weeks. She said she could either give him $10 per week, or she could give him $2 the first week and then double the amount each week. Write a number pattern for Justin's weekly earnings under the second plan. Which plan should Justin choose?

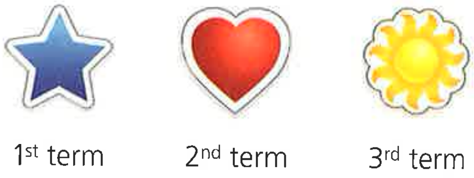
Show your work.

Answer ______________________________

Independent Practice

Solve the problems.

MP7 **21.** A roll of stickers starts with the three shapes shown below. The shapes repeat to make up the entire roll.



Which shape will be the hundredth sticker in the roll?

Show your work.

Answer ______________________________

MP8 **22.** Simon said, "If the rule for a pattern is *add 2*, then the terms will either be all even numbers or all odd numbers." Is Simon correct?

Answer ______________________________

Justify your answer using words, drawings, or numbers.

MP7 **23.** The rule for the number pattern below is *add 8*, starting with 8.

8, 16, 24, 32, 40, . . .

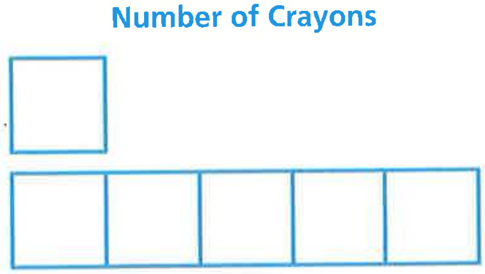
Find the 50th term of this pattern without finding all the terms in between.

Answer ______________________________

Justify your answer using words, drawings, or numbers.

UNIT 1 Common Core Review

1. Odell has 5 times as many crayons as Jason. Jason has 8 crayons. Label the model to show how many crayons each person has.



For exercises 2 and 3, use the model above.

2. How many crayons does Odell have?

Answer ______________________________

3. Which multiplication equation correctly compares the number of crayons the two boys have?

a. $4 \times 8 = 32$

b. $5 \times 8 = 40$

c. $2 \times 10 = 40$

d. $6 \times 8 = 48$

For exercises 4–6, circle the correct answer or answers.

4. Which of these numbers are multiples of 6?

a. 10

b. 16

c. 30

d. 48

5. At the library sale, a DVD costs $12. That is 3 times the cost of a book. Which equation can you use to find the cost of a book?

a. $3 \times 12 = n$

b. $12 \times n = 3$

c. $3 \times n = 12$

d. $12 \times 3 = n$

6. A small box holds 6 toy cars. A large box holds 18 toy cars. How many times as many cars can the large box hold as the small box?

a. 2 times as many

b. 3 times as many

c. 4 times as many

d. 6 times as many

For exercise 7, use the rule to generate the pattern. Then write at least two observations about the pattern.

MP8 **7.** Start with 3. Add 6.

3, ____, ____, ____, ____, ____

Observations

MP3 **8.** Chrissy estimates that if you add 20 + 31, then divide the sum by 7, the answer is about 12. Is this a reasonable estimate? Explain.

For exercises 9 and 10, find all the factors. Tell whether the number is *prime* or *composite*.

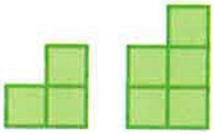
9. 56

prime or ***composite***? ____________

10. 61

prime or ***composite***? ____________

11. Draw the next two shapes in the pattern. Name the kind of pattern. Then write its rule.



This is a ____________ shape pattern.
growing or repeating

Rule ____________

Solve the problems.

MP1 **12.** Melinda is 3 times as old as Tom. Vihn is 4 times as old as Melinda. If Melinda is 9 years old, how old are Tom and Vihn?

Show your work.

Answer ____________

UNIT 1 Common Core Review

MP2 **13.** Franklin is packing his old toy trucks into boxes. He has 22 trucks. If each box holds 4 trucks, how many boxes will Franklin need?

Show your work.

Answer ____________________

MP7 **14.** The rule for the number pattern below is to add 5, starting with 5. What is the hundredth number in the pattern?

5, 10, 15, 20, 25, . . .

Show your work.

Answer ____________________

MP1 **15.** For the school carnival, Ms. Chen wants to bag 84 marbles to give away as prizes. How many ways can Ms. Chen bag the marbles so that:

- each bag has at least 3 marbles
- each bag has no more than 10 marbles
- all bags have the same number of marbles?

Show your work.

Answer ____________________

Progress Check

UNIT 2

Look at how the Common Core standards you have learned and will learn connect.

It is very important for you to understand the standards from the prior grade level so that you will be able to develop an understanding of number and operations in base ten in this unit and be prepared for next year. To practice your skills, go to sadlierconnect.com.

GRADE 3		GRADE 4		GRADE 5
I Can...	Before Unit 2	**Can I ?**	After Unit 2	**I Will...**
	☐	**4.NBT.1** Understand place value in whole numbers	☐	**5.NBT.1** Understand place value in whole numbers and decimal numbers **5.NBT.2** Explain patterns of zeros when multiplying by powers of 10 Use whole number exponents to show powers of 10
	☐	**4.NBT.2** Read and write whole numbers using numerals, number names, and expanded form	☐	**5.NBT.3** Read, write, and compare decimals to thousandths
	☐	Compare whole numbers	☐	
3.NBT.1 Round whole numbers to the nearest 10 or 100	☐	**4.NBT.3** Round multi-digit whole numbers to any place	☐	**5.NBT.4** Round decimals to any place
3.NBT.2 Add and subtract whole numbers within 1000	☐	**4.NBT.4** Add and subtract whole numbers within 1,000,000	☐	**5.NBT.7** Add and subtract decimals
3.NBT.3 Multiply one-digit numbers by multiples of 10	☐	**4.NBT.5** Multiply whole numbers by one-digit whole numbers	☐	**5.NBT.5** Multiply whole numbers
3.OA.7 Multiply and divide whole numbers within 100	☐	Multiply two 2-digit numbers	☐	**5.NBT.7** Multiply and divide decimals
	☐	**4.NBT.6** Divide whole numbers by one-digit divisors	☐	**5.NBT.6** Divide whole numbers by two-digit divisors

HOME ✦ CONNECT...

Problem solving is an important skill for your child to master. Your child will make use of various problem-solving methods to solve word problems involving addition, subtraction, multiplication, and division. Support your child by using the following problem-solving model:

- **Read** Read the problem with your child. Focus on the facts and the questions. Ask: *What facts do you know? What do you need to find out?*
- **Plan** Outline a plan with your child. Plan how to solve the problem. Ask: *What operation* (addition, subtraction, multiplication, or division) *will you use? Do you need to use 1 step or 2 steps? Will you draw a picture? How have you solved similar problems?*
- **Solve** Follow the plan to solve the problem with your child. Ask: *Did you answer the question? Did you label your answer?*
- **Check** Test that the solution is reasonable. Ask: *How can you solve the problem a different way? Is the answer the same? How can you estimate to check your answer?*

Conversation Starter: Your child will need to have an understanding of both place value and operations. Practice place value by naming numbers for your child to write. Discuss the place value of each digit in the number. For example, the standard form of *seven thousand six hundred and seventy two* is 7,672. It has seven thousands, six hundreds, seven tens, and two ones.

In this unit your child will:

- Understand place value of whole numbers.
- Read, write, and compare whole numbers.
- Apply place value understanding to round whole numbers.
- Add and subtract fluently with whole numbers.
- Multiply and divide whole numbers using place value and properties of operations.

NOTE: All of these learning goals for your child are based on the Grade 4 Common Core State Standards for Mathematical Content.

Ways to Help Your Child

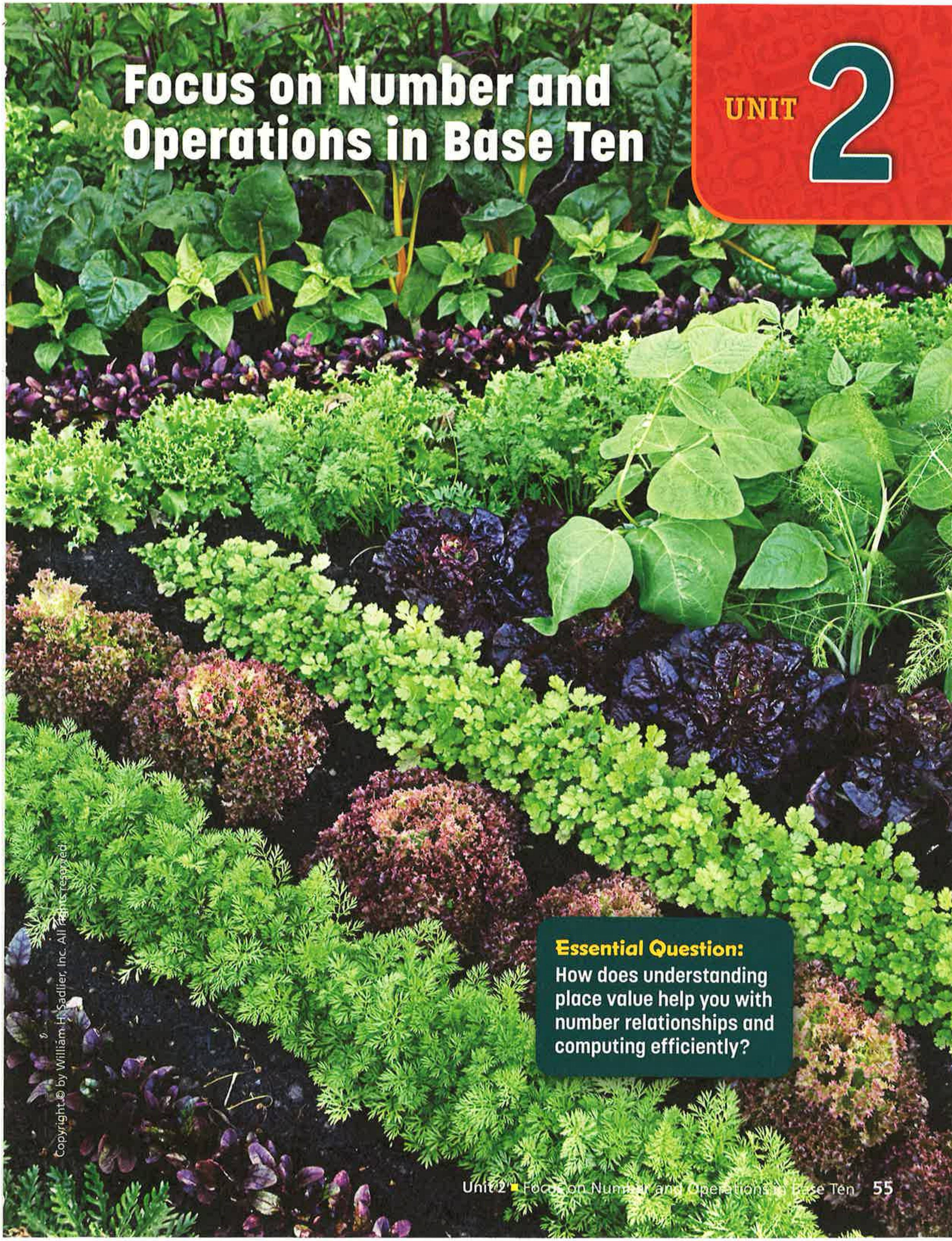
Make time to talk with your child's teacher about your child's level of progress. Discuss ways that you can assist with your child's learning at home. If your child needs extra practice, your support can really make a difference.

ONLINE

For more Home Connect activities, continue online at sadlierconnect.com

UNIT 2

Focus on Number and Operations in Base Ten



Essential Question:
How does understanding place value help you with number relationships and computing efficiently?

Lesson

6 Understand Place Value of Whole Numbers

Essential Question: How does the place of a digit in a number affect its value?

4.NBT.1

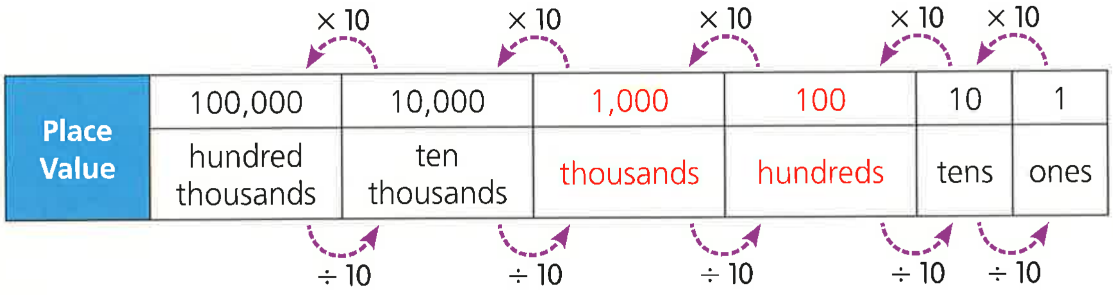
Guided Instruction

In this lesson you will learn about place value relationships in whole numbers.

Understand: Place value and the value of a digit

Kevin sees the digit 2 twice in the number 162,257. What is the value of each digit 2 in 162,257?

To find the value of each digit 2, use a place-value chart. The value of each digit in a whole number is shown by its place in the number. The value of the digit in each place is ten times greater than its value in the place to the right.



Use the place-value chart below to find the value of each 2 in 162,257.

hundred thousands	ten thousands	thousands	hundreds	tens	ones
1	**6**	**2**	**2**	**5**	**7**

To find the value of each digit 2, multiply it by its place value.

The place-value chart shows 2 in the thousands and hundreds places.

2 thousands = 2 × 1,000 = 2,000
2 hundreds = 2 × 100 = 200

Then 2,000 is ten times the value of 200, or
2,000 = 10 × 200

In 162,257, the value of the digit 2 in the thousands place is 2,000 and the value of the digit 2 in the hundreds place is 200.

- Show the relationship between 2,000 and 200 using division.

Guided Instruction

Understand: Place value and multiples of 10

A mistake in a store ad shows the price of a digital notepad as $30. The correct price of the digital notepad is $300. How many times as much as $30 is $300?

To find how many times as much $300 is compared to $30, you can use place value. The value of the hundreds place is 10 times the value of the tens place: 100 = 10 × 10.

Remember!
All numbers that end with 0 are multiples of 10.

hundreds	tens	ones
3	**0**	**0**
	3	**0**

30 = 3 tens

300 = 10 × 3 tens = 3 hundreds

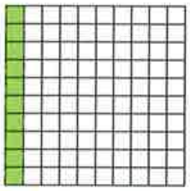
300 = 10 × 30

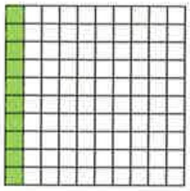
➡ $300 is ten times as much as $30.

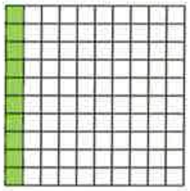
Because 30 and 300 are multiples of 10, you can use a model to find the number of tens in 300.

Use place-value models to show that 300 is the same as 30 × 10.

$$\begin{aligned} 300 &= 3 \times 100 \\ &= 3 \times (10 \times 10) \longrightarrow \\ &= (3 \times 10) \times 10 \\ &= 30 \times 10 \end{aligned}$$


10 × 10


10 × 10


10 × 10

What number times 30 would move the digit 3 to the thousands place in a place-value chart? Explain your answer.

Guided Instruction

Connect: Relationships between place values

Abbie says that if you divide the value of each digit in 4,444 by the value of the digit to the right, the quotient will always be 10. Is Abbie correct?

Use place value.

Step 1

Start with the digits in the tens and the ones places.

4, 4 4 4.

40 ÷ 4 = 10.

Step 2

Continue with the digits in the hundreds and in the tens places.

4, 4 4 4.

400 ÷ 40 = 10.

Step 3

Then continue with the digits in the thousands and in the hundreds places.

4, 4 4 4.

4,000 ÷ 400 = 10.

Yes, Abbie is correct. If you divide the value of each digit in Abbie's number by the value of the digit to the right, the quotient will always be 10.

Does this pattern continue for 444,444? Explain.

Guided Practice

Complete the place-value chart. Write the missing values and place names.

1.

Place Value	100,000		1,000		10	1
	hundred thousands	ten thousands			tens	ones

Complete the sentences for each underlined digit. You can use the place-value chart above to help.

2. 450,32**8** The digit 8 is in the ____________ place.

Its value is 8 × 1 = ____.

3. 450,3**2**8 The digit 2 is in the ____________ place.

Its value is ____ × 10 = 20.

4. 450,**3**28 The digit 3 is in the ____________ place.

Its value is 3 × ____________ = ____________.

5. 45**0**,328 The digit 0 is in the thousands place.

It shows that there are ____________ thousands.

6. 4**5**0,328 The digit 5 is in the ____________ place.

Its value is 5 × ____________ = 50,000.

7. **4**50,328 The digit 4 is in the hundred thousands place.

Its value is ____ times the value of ____________.

Think • Pair • Share

MP4 **8.** Use the place-value chart in exercise 1. Describe the rule for the pattern shown by the place values from right to left, starting with 1. How would you extend the place-value chart to show 1,000,000, or one million?

Independent Practice

Write 546,893 in the place-value chart. Then name the value of each digit in the number.

hundred thousands	ten thousands	thousands	hundreds	tens	ones

1. ____ ones = 3

2. ____ tens = ____

3. ____ hundreds = ________

4. ____ thousands = ________

5. ____ ten thousands = ________

6. ____ hundred thousands = ________

Write the place for the underlined digit. Then write the value of the digit.

7. 7,**4**45 place: ____________________

The digit 4 represents ____________.

8. 16,2**3**8 place: ____________________

The digit 3 represents ____________.

9. 4**8**,524 place: ____________________

The digit 8 represents ____________.

10. **4**01,902 place: ____________________

The digit 4 represents ____________.

Compare the values of the digits in the given place. Circle the number whose digit has the greater value.

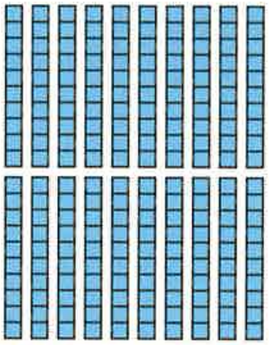
11. tens 572 614

12. thousands 14,308 22,563

13. ten thousands 263,450 275,029

Independent Practice

For exercises 14–16, use the model at the right.



14. The model shows ____ tens.

15. The model shows ____ hundreds.

16. There are ____ times as many tens as hundreds.

Write the value of the underlined digit.
Then regroup the value.

17. $\underline{3}49$ __________ = ____ tens

18. $\underline{2},158$ __________ = ____ hundreds

19. $\underline{1}6,331$ __________ = ____ thousands

20. Look at the number shown below. Under each digit, write the letter of the matching description.

6 6 8 , 8 8 4

____ ____ ____ ____ ____ ____

- **a.** The number of ones
- **b.** Has a value of 80
- **c.** Represents 10 times the value of the tens
- **d.** The number of hundred thousands
- **e.** Has a place value 10 times the value of the thousands place
- **f.** Represents 10 times the value of the hundreds

21. 80 × 10 = __________

22. 600 × 10 = __________

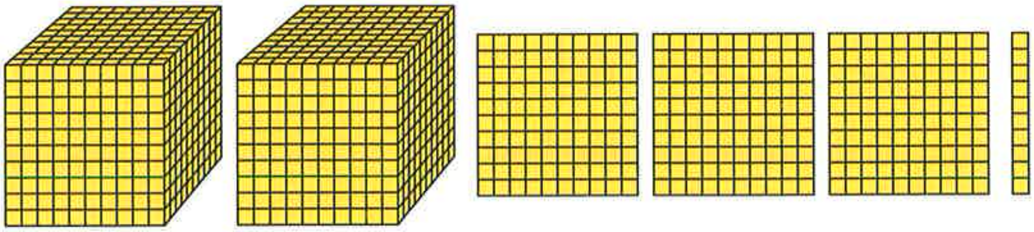
23. 5,000 ÷ 500 = __________

24. 7,500 ÷ 750 = __________

Independent Practice

MP6 **25.** John makes a mistake when he inputs the number 1,205. He inputs the number 2,105 instead. Use place value to explain John's mistake.

MP1 **26.** What number is represented by the model? Explain the value of each digit in the number.



Solve the problems.

MP7 **27.** A cashier counts 25 ten-dollar bills and writes the total amount of money as a number. What is this number, and what is the place of the digit 2?

Show your work.

Answer ____________________

MP2 **28.** Tanya says that 36,000 is the same as 360 hundreds. Is Tanya correct?

Show your work.

Answer ____________________

Independent Practice

MP6 **29.** Samantha is taking $2,000 out of her savings. She asks to have the money in 20-dollar bills. How many 20-dollar bills should Samantha receive?

Show your work.

Answer ________________

MP7 **30.** Alejandro and Erin estimate the number of people at a football game. Alejandro thinks there are about 60,000 people in the stadium. Erin says there are about 6,000 people. How many times as much is Alejandro's estimate as Erin's estimate?

Show your work.

Answer ________________

MP8 **31.** Briana says that $1,200 \div 12 = 10$. Is Briana's answer correct?

Answer ________________

Justify your answer using words, drawings, or numbers.

MP6 **32.** Nicole and Micah are talking about the number 12,459. Micah says that the value of the thousands digit is 10 times the value of the hundreds digit. Nicole says this is not true, since the two digits are not the same. Who is correct?

Answer ________________

Justify your answer using words, drawings, or numbers.

Lesson

7 Read, Write, and Compare Whole Numbers

Essential Question:
How do you use place value to name and compare numbers?
4.NBT.2

Words to Know:
period
expanded form

Guided Instruction

In this lesson you will learn how to read, write, and compare whole numbers.

Understand: Names for whole numbers

Kendra is reading her report about Mount Everest aloud. The height of Mount Everest is 29,035 feet. How should Kendra read this number?

To find how Kendra should read the number, write the number 29,035 in a place-value chart.

To read a number of thousands or greater, read each group of digits. A period is a group of 3 places. For example in the chart below, the periods are the thousands period, and the ones period. A comma usually separates periods.

Thousands			Ones		
hundreds	tens	ones	hundreds	tens	ones
	2	**9**	**0**	**3**	**5**

Read the total number of thousands written before the comma or in the thousands period:

numeral	number name
29,035	*twenty-nine thousand*

The ones period groups the values less than 1,000. Read the total value of the ones group of digits.

29,035	*thirty five*

Remember!
You don't need to say or write "ones" for a number less than 1,000.

Kendra can read 29,035 as *twenty-nine thousand, thirty five.*

Write the numeral for *six hundred fifteen thousand, two hundred one* and the numeral for *fifty-two thousand, forty-nine.*

Guided Instruction

Understand: Numbers in expanded form

You can think about the height of Mount Everest in another way. Use the expanded form to see the value of each digit in 29,035.

You can also write a number in expanded form. The expanded form of a number shows the value of each digit. Find the value of each digit in 29,035.

$$\begin{array}{cccccc} 2 & 9 & , & 0 & 3 & 5 \\ \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow \\ 20{,}000 & 9{,}000 & & 0 & 30 & 5 \end{array}$$

Remember!
Multiply a digit by its place value to find its value.

29,035 = 20,000 + 9,000 + 30 + 5

The expanded form of 29,035 is 20,000 + 9,000 + 30 + 5.

Understand: Comparisons of whole numbers

The height of Mount McKinley is 20,320 feet. Which mountain is taller, Mount McKinley or Mount Everest?

To compare numbers, first line up the digits by place value. You can use a place-value chart to help.

	Thousands			Ones		
	hundreds	tens	ones	hundreds	tens	ones
Mt. Everest		2	9	0	3	5
Mt. McKinley		2	0	3	2	0

Compare the digits in each place, starting with the greatest place.
Stop in the first place where the digits are different.
Ten thousands: 2 = 2
Thousands: 9 > 0

29,035 has the greater number of thousands, so 29,035 > 20,320.

Remember!
= means *is equal to.*
< means *is less than.*
> means *is greater than.*

Mount Everest is taller than Mount McKinley.

Guided Instruction

Connect: Represent and compare whole numbers

A radio announcer says that two hundred fifteen thousand, four hundred seven people lost power in a storm. A news article reported that 215,470 people lost power. How do these numbers compare?

Write the numbers in the same form and then compare them.

Step 1

Write this number name in standard form:
two hundred fifteen thousand, four hundred seven.

Write the total number of thousands: 215.
Write the total number of ones given after the comma: 407.
Now write the full number with a comma: 215,407.
215,407 is not the same as 215,470, the number from the news article.

Step 2

To see why the numbers are different, write them in expanded form.
Find and list the values of the digits in each number.

215,407 $200{,}000 + 10{,}000 + 5{,}000 + 400 + 0 + 7$
215,470 $200{,}000 + 10{,}000 + 5{,}000 + 400 + 70 + 0$

There are different numbers of tens and ones.

Step 3

Compare the numbers. Line up the digits by place value and stop in the first place where the digits are different.

215,407 0 tens is less than 7 tens: $0 < 7$.
215,470

215,407 has fewer tens than 215,470.

▶ $215{,}407 < 215{,}470$.

Read aloud 215,470, the number from the news article. Write the number name.

Guided Practice

Fill in the blanks to write the number in a different form.

1. 21,109

number name: __________-one __________, one __________ nine

2. 492,013

number name: __________ hundred __________-__________

thousand, __________

3. sixty-three thousand, eight hundred fifty two

numeral: ____ ____, ____ ____ ____

4. five hundred fourteen thousand, three hundred three

numeral: __________

5. 89,780

expanded form: __________ + 9,000 + __________ + ____

6. 307,326

expanded form: __________ + __________ + __________ + ____ + ____

Write < , =, or > to compare the numbers.

7. 6,**5**89 ____ 6,**9**28

8. 2**1**,807 ____ 2**0**,931

9. 37,146 ____ 37,146

10. 458,923 ____ 459,823

Think • Pair • Share

MP1 **11.** Extend the place-value chart on page 64 to show 21,385,604. Be sure to label the millions period. Explain the value of each digit.

Independent Practice

Fill in the blanks to write each number in three forms. Look at all three forms to find the information you need.

1. 68,223

sixty-__________ thousand, two __________ twenty three

__________ + 8,000 + 200 + ____ + 3

2. 104, ____ ____ ____

one hundred __________ __________, eight hundred __________

__________ + 4,000 + __________ + 10 + 6

3. 98 ____, ____ 31

__________ __________ eighty-five __________, thirty one

__________ + __________ + 5,000 + ____ + 1

Circle the correct answer.

4. Which is the number name for 712,358?

a. seven hundred thousand twelve, three fifty eight

b. seventy one thousand, twenty-three hundred fifty-eight

c. seven hundred twelve thousand, three hundred fifty eight

d. seven hundred twelve thousand, thirty-five hundred eight

5. Which of these is a different number from the others?

a. 42,211

b. 40,000 + 2,000 + 200 + 1

c. 4 ten thousands + 2 thousands + 2 hundreds + 1 one

d. forty-two thousand, two hundred one

6. Which of these shows one million?

a. 10,000

b. 100,000

c. 1,000,000

d. 100,000,000

Independent Practice

Write the given number in two other forms.

7. 33,410

expanded form: ______ + ______ + ______ + ___

number name: ______________________

8. 200,000 + 80,000 + 4,000 + 90 + 9

numeral: ______

number name: ______________________

Name the first place where the digits in both numbers are different. Then write < , =, or > to compare the numbers.

9. **4**9,603
57,213

____

49,603 ___ 57,213

10. 64,**8**19
64,**5**47

____

64,819 ___ 64,547

11. 12,213
9,547

____

12,213 ___ 9,547

12. 197,687
861,070

____

861,070 ___ 197,687

Write < , =, or > to compare the numbers. Circle the first digit that is different in the number pairs.

13. 3,526 ___ 3,526

14. 99,287 ___ 89,999

15. 28,843 ___ 28,871

16. 740,489 ___ 739,501

Independent Practice

Use place values in your explanation.

MP3 **17.** Colin is taking notes in class. The teacher says the number *eighty-five thousand, sixty two*. Colin writes *85,602* in his notebook. Explain the mistake Colin made.

MP6 **18.** Mara says that 28,597 is greater than 101,200 because 2 is greater than 1. Is Mara correct? Explain your answer.

Solve the problems.

MP8 **19.** James counts the number of times his heart beats in one minute. He figures out that his heart beats about 125,280 times in one day. What is 125,280 in expanded form?

Show your work.

Answer ______________________

MP7 **20.** Lucia says that sixteen thousand, two hundred seventeen has fewer tens than sixteen thousand, one hundred forty eight. What is the difference in the number of tens for these numbers?

Show your work.

Answer ______________________

Independent Practice

For exercises 21–24, use the table at the right.

Land Areas of Four States	
State	**Land Area (square miles)**
Arizona	113,634
California	155,959
Colorado	103,717
Kansas	81,815

MP2 **21.** Humberto writes the area of each state shown in the table in expanded form. Which two states have the same number of thousands in their expanded forms?

Show your work.

Answer ______

MP6 **22.** Laurel reads the area of Arizona as *one hundred three thousand, six hundred thirty four* square miles. Which area has a greater number of ten thousands, Laurel's or the area in the table?

Show your work.

Answer ______

MP7 **23.** Max knows that one state definitely has the least area, just by looking at the table. Which state is Max thinking of?

Answer ______

Justify your answer using words, drawings, or numbers.

MP2 **24.** After Max identifies the state with the least area, he compares the areas of the three other states shown. Which place in the numbers will tell Max which state has the greatest area?

Answer place ______

state ______

Justify your answer using words, drawings, or numbers.

Lesson

8 Apply Place Value to Round Whole Numbers

Essential Question:
How can place value help you round to different places in a whole number?
4.NBT.3

Guided Instruction

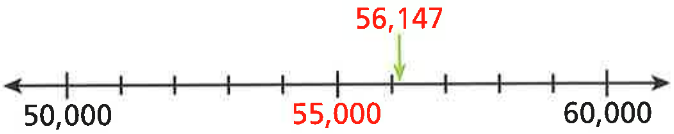
In this lesson you will learn how to round to any place in a whole number.

Understand: The numbers you use to round

In one day, 56,147 people watch a video of the baby panda at the zoo. Lily says that 56,147 is about 60,000 people. Is this a reasonable estimate?

To find an estimate for 56,147, you can round the number.

The greatest place in 56,147 is ten thousands. Between which two ten thousands is 56,147 the closest? You can make a number line to help.



50,000 < 56,147 < 60,000

To find which ten thousand 56,147 is nearer to, you can use the number line or compare the digits.

Use the number line:
Mark the number that is exactly halfway between 50,000 and 60,000: 55,000. On the number line, 56,147 is to the right of 55,000, the halfway mark. Since 56,147 is nearer to 60,000 than to 50,000, round 56,147 up to 60,000.

Compare the digits: 56,147 6 > 5
55,000

Look at the digit to the right of the place to which you are rounding, in this case the thousands place. If it is 5 or greater, then round up.

Since 6 thousands is greater than 5 thousands, 56,147 is greater than 55,000. Then 56,147 is nearer to 60,000 than to 50,000.

Yes. 60,000 is a reasonable estimate for 56,147 people.

What place can you round 56,147 to for a closer estimate?

Guided Instruction

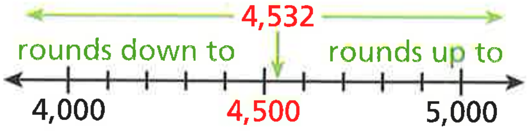
Connect: What you know about rounding and closer estimates

The zoo gives a free panda poster to visitors each Saturday. Last Saturday, 4,532 people visited the zoo. If approximately the same number of people visit each Saturday, estimate the number of posters the zoo should have ready to give out this Saturday.

You can use place value to round a number to different places.

Step 1

For a first estimate, look at the digit in the greatest place in the number.



The 4 in 4,532 is in the thousands place. Round to the nearest thousand.

$4{,}000 < 4{,}532 < 5{,}000$

To find which thousand 4,532 is nearer to, compare it to the halfway mark. The number that is halfway between 4,000 and 5,000 is 4,500.

Step 2

To round to the nearest thousand, compare the hundreds digit in 4,532 to the hundreds digits in 4,500; $5 = 5$

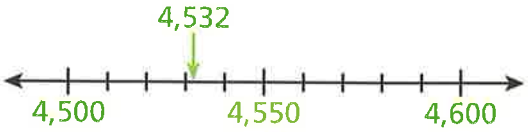
4,532 has the same number of hundreds as 4,500, the number halfway between 4,000 and 5,000.

So 4,532 rounds up to 5,000.

Remember!
If the digit to the right of the place to which you are rounding is 5 or greater, round up.

Step 3

To find a closer estimate, you can round 4,532 to the hundreds place.



The next closest hundred to 4,500 is 4,600. 4,550 is the number halfway between 4,500 and 4,600.

To round to the nearest hundred, compare the tens digit in 4,532 to the tens digit in 4,550: $3 < 5$

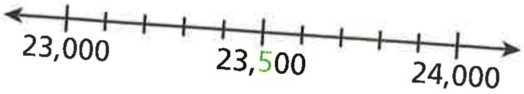
So rounding to the nearest hundred, 4,532 rounds down to 4,500.

➡ Depending on which place value you estimated to, the zoo should have about 5,000 or 4,500 posters ready to give out on Saturday.

Guided Practice

Round each number to the underlined place. You can use the number line to help.

1. 23,485



3 is in the ____________ place.

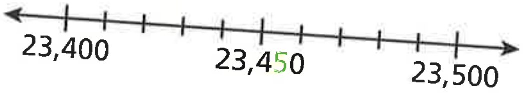
The closest thousands are ____________ and 24,000.

Compare the hundreds digit in 23,485 to the 5 hundreds in ____________.

4 ______ 5

23,485 rounds ____________ to ____________.

2. 23,485



4 is in the ____________ place.

____________ < 23,485 < ____________

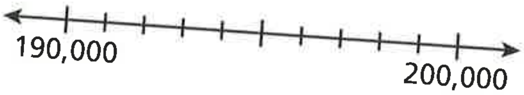
Compare the ____________ digit in 23,485 to the ____ tens in ____________.

8 ______ 5

23,485 rounds ____________ to ____________.

For exercises 3–5, use the number line at the right.

3. What number is exactly halfway between 190,000 and 200,000?



____

4. Write four numbers that can be rounded up to 200,000.

____________ ____________ ____________ ____________

5. Write four numbers that can be rounded down to 190,000.

____________ ____________ ____________ ____________

Guided Practice

Round each number to the underlined place. Write the numbers you can round to. Then circle the correct rounded number.

6. $\_\_\_\_\_\_\_\_ < \underline{6},742 < 7,000$

7. $\_\_\_\_\_\_\_\_ < \underline{1}2,099 < 20,000$

8. $\_\_\_\_\_\_\_\_ < 3\underline{8},250 < 39,000$

9. $\_\_\_\_\_\_\_\_ < 5\underline{4}5,123 < \_\_\_\_\_\_\_\_$

For exercises 10 and 11, you can use a number line.

10. Ross is going to Australia to visit his grandparents. The airline says the distance between Boston and Brisbane is 9,773 miles. What is a reasonable estimate for this distance? Explain your answer.

11. Audrey read that the area of a new shopping mall is 57,600 square feet. What are the greatest and least possible actual areas of the mall if 58,000 square feet is an estimate to the nearest thousand?

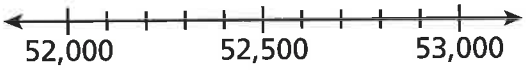
Think • Pair • Share

MP4 12. Toni draws a number line and uses it to round 83,768 to the nearest ten thousand. Then she decides to round 83,768 to the nearest thousand. Show how Toni can use the same number line to do this?

Independent Practice

Round the number to the underlined place. You can use the number line to help.

1. 52,143



2 is in the ____________ place.

The closest thousands are ____________ and ____________.

Compare the ____________ digit in 52,143 to the ____________

in ____________.

1 ____ 5

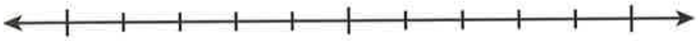
52,143 rounds ____________ to ____________.

Round each number to the underlined place. Label the number line to show the numbers you are using.

2. 3,325

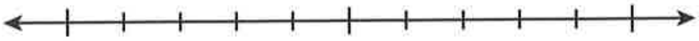
Answer ____________

3. 45,918



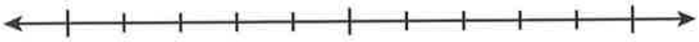
Answer ____________

4. 67,472



Answer ____________

5. 208,366



Answer ____________

Independent Practice

Match the number with the estimate rounded to the underlined place. Write the letter of the correct estimate.

6. **7**16,549 ____

7. 7**1**6,549 ____

8. 71**6**,549 ____

9. 716,**5**49 ____

Estimates

a. 717,000

b. 720,000

c. 700,000

d. 716,500

Round each number to the given place.

10. 10,401 to the thousands place ________

11. 26,792 to the ten thousands place ________

12. 48,638 to the thousands place ________

13. 84,151 to the hundreds place ________

14. 447,950 to the hundred thousands place ________

15. 625,099 to the ten thousands place ________

16. 773,646 to the thousands place ________

17. 998,979 to the hundreds place ________

Solve the problems.

18. A restaurant owner is ordering supplies for the week. Last week, she ordered 1,458 pounds of potatoes. Which is the best estimate for this number?

a. 1,000 pounds **b.** 1,400 pounds

c. 1,500 pounds **d.** 2,000 pounds

19. Chung's family is hiking all 2,184 miles of the Appalachian Trail. He estimates the trail length as 2,000 miles. Which estimate will better help the family make sure that they bring enough supplies?

a. 1,000 miles **b.** 2,100 miles

c. 2,200 miles **d.** 3,000 miles

Independent Practice

MP5 **20.** The local news says that ticket sales for a new movie were $256,500 on Friday night, $421,000 on Saturday, and $301,850 on Sunday. Do you think these numbers are estimates or exact amounts? Explain.

MP6 **21.** Does it make sense to round a six-digit number to the tens place? Explain your answer with an example.

Solve the problems.

MP2 **22.** At a carnival, Jenn correctly guesses the exact number of pennies in a jar as 124,983. The closest guess the day before was 124,983 rounded to the hundreds place. What was this guess?

Show your work.

Answer ______________________________

MP8 **23.** A store owner is adding $5,641 and $5,238 to find his total sales for the past two days. He rounds each amount to the nearest hundred dollars and then adds the rounded numbers. What is the store owner's estimated total sales amount?

Show your work.

Answer ______________________________

Independent Practice

MP1 **24.** Victor estimates the sum of 289 and 532 in two ways. First he rounds both numbers and then adds them. Then he adds the exact numbers and rounds the sum. Both methods give him the same final number, but he gets a different answer when he rounds to a different place. What two estimated sums does Victor find?

Show your work.

Answer ______________________

MP7 **25.** Ben and Lisa round the same number to different places. Ben rounds to the nearest ten thousand and gets 70,000. Lisa rounds to the nearest thousand and gets 68,000. If they both rounded correctly, what are the greatest and least possible numbers they could have rounded?

Show your work.

Answer ______________________

MP3 **26.** An Arctic tern can fly more than 43,496 miles as it migrates each year. Round this number to the nearest ten thousand miles. Then round 43,496 miles to the place of your choice for a better estimate.

Answer ______________________

Justify your answer using words, drawings, or numbers.

MP8 **27.** Owen incorrectly rounds 384,478 to the thousands place by rounding in three steps. He rounds to the nearest ten first, and then rounds two more times until he reaches 385,000. Is it possible that Owen rounded correctly at each step, even though his final estimate is wrong?

Answer ______________________

Justify your answer using words, drawings, or numbers.

Lesson

9 Add and Subtract Fluently with Whole Numbers

Essential Question:
How can you add and subtract whole numbers efficiently?
4.NBT.4

Guided Instruction

In this lesson you will learn the standard algorithms for adding and subtracting using place value.

Understand: Place value and addition

One report states that scientists have discovered 307,674 different kinds of plants and 64,283 kinds of vertebrates, or animals with a backbone. How many different kinds of plants and vertebrates are there altogether?

To find the sum of 307,674 and 64,283, you can add. Align the digits by place value.

Remember!
You can only write one digit for each place of the sum.

Add the ones: 4 + 3 = 7.
Add the tens: 7 + 8 = 15.
15 tens is the same as
1 hundred 5 tens.
Regroup 15 tens as 1 hundred 5 tens.
Write the 1 hundred above the hundreds place, and write the 5 tens in the sum.

$$\begin{array}{r} {}^{1} \\ 307{,}674 \\ +\ \ 64{,}283 \\ \hline 57 \end{array}$$

Keep adding the digits in each place, regrouping whenever the sum for that place is 10 or greater.

Add the hundreds: 1 + 6 + 2 = 9
Add the thousands: 7 + 4 = 11.
11 thousands is the same as
1 ten thousands 1 thousand.
Regroup 11 thousands as 1 ten thousand
1 thousand.

$$\begin{array}{r} {}^{1}\ \ {}^{1} \\ 307{,}674 \\ +\ \ 64{,}283 \\ \hline 1{,}957 \end{array}$$

Add the ten thousands: 1 + 0 + 6 = 7
There is nothing to add to the 3 hundred thousands, so write 3 in the sum.

$$\begin{array}{r} {}^{1}\ \ {}^{1} \\ 307{,}674 \\ +\downarrow 64{,}283 \\ \hline 371{,}957 \end{array}$$

➡ There are 371,957 different kinds of plants and vertebrates altogether.

Guided Instruction

Understand: Subtraction and regrouping

Spiders, scorpions, and other animals with eight legs are called *arachnids*. The report also states that there are 102,248 kinds of arachnids. How many more kinds of arachnids are there than the 64,283 kinds of vertebrates?

To find how many more kinds of arachnids there are, start by estimating the difference. This estimate will allow you to judge the reasonableness of your exact answer.

102,248 − 64,283
100,000 − 64,000 = 36,000

Next align the digits by place value. Subtract in each place.

Start with the ones: 8 − 3 = 5.
To subtract 4 tens − 8 tens, more tens are needed. Regroup.

$$\begin{array}{rrrrrrr} & 1 & 0 & 2, & 2 & 4 & 8 \\ - & & 6 & 4, & 2 & 8 & 3 \\ \hline & & & & & ? & 5 \end{array}$$

Regroup 2 hundreds 4 tens as 1 hundred 14 tens.
Then subtract: 14 − 8 = 6.

$$\begin{array}{rrrrrrr} & & & & 1 & 14 & \\ & 1 & 0 & 2, & \cancel{2} & \cancel{4} & 8 \\ - & & 6 & 4, & 2 & 8 & 3 \\ \hline & & & & & 6 & 5 \end{array}$$

Since you must subtract 1 hundred − 2 hundreds, regroup the thousands:
2 thousands 1 hundred as 1 thousand 11 hundreds.
Then subtract the hundreds. 11 − 2 = 9

$$\begin{array}{rrrrrrr} & & & 1 & 11 & 14 & \\ & 1 & 0 & \cancel{2}, & \cancel{2} & \cancel{4} & 8 \\ - & & 6 & 4, & 2 & 8 & 3 \\ \hline & & & & 9 & 6 & 5 \end{array}$$

To subtract 1 thousand − 4 thousands, regroup ten thousands:
10 ten thousands 1 thousand as 9 ten thousand 11 thousands.
Subtract the thousands. 11 − 4 = 7

$$\begin{array}{rrrrrr} & 9 & 11 & 11 & 14 & \\ & \cancel{10} & \cancel{2}, & \cancel{2} & \cancel{4} & 8 \\ - & 6 & 4, & 2 & 8 & 3 \\ \hline & & 7, & 9 & 6 & 5 \end{array}$$

Now subtract the ten thousands. 9 − 6 = 3

$$\begin{array}{rrrrrr} & 9 & 11 & 11 & 14 & \\ & \cancel{10} & \cancel{2} & \cancel{2} & \cancel{4} & 8 \\ - & 6 & 4, & 2 & 8 & 3 \\ \hline & 3 & 7, & 9 & 6 & 5 \end{array}$$

37,965 is close to the estimate of 36,000 so the exact answer is reasonable.

➡ There are exactly 37,965 more kinds of arachnids than vertebrates.

Guided Instruction

Connect: Subtraction and addition

Sometimes scientists disagree over their findings. One report says that there are 310,129 kinds of plants instead of 307,674. What is the difference between the number of plants?

Subtract to find the difference between the number of plants. Then add to check.

Step 1

To subtract, align the digits by place value.
Subtract the ones: 9 − 4 = 5.
Regroup hundreds and tens.
Subtract the tens.

$$\begin{array}{r} 0\;12 \\ 310,\cancel{1}\cancel{2}9 \\ -\,307,674 \\ \hline 55 \end{array}$$

Regroup thousands and hundreds as needed.
Subtract the hundreds.

$$\begin{array}{r} 0\;9\;10\,12 \\ 3\cancel{1}\cancel{0},\cancel{1}\cancel{2}9 \\ -\,307,674 \\ \hline 455 \end{array}$$

Remember!
The expanded form of a number shows the values of all its digits.
310,129 = 300,000 + 10,000 + 100 + 20 + 9

Subtract the thousands.
Subtract the ten thousands.
Subtract the hundred thousands.
310,129 − 307,674 = 2,455

$$\begin{array}{r} 0\;9\;10\,12 \\ 3\cancel{1}\cancel{0},\cancel{1}\cancel{2}9 \\ -\,307,674 \\ \hline 2,455 \end{array}$$

Step 2

You can add to check your answer.
If the sum is the number you started with, then you subtracted correctly.

$$\begin{array}{r} 1\;1\;1 \\ 307,674 \\ +\;\;\;2,455 \\ \hline 310,129 \end{array}$$

Remember!
Addition and subtraction are inverse operations.

➡ The difference between the number of plants is 2,455.

✏ You know that 10,000 − 1 = 9,999. Show the regrouping that is needed to find the answer.

Guided Practice

For each step in exercises 1–4, write the missing digits to add or subtract.

1.

$$\begin{array}{r} \square \\ 3,\ 5\ 8\ 4 \\ +\,2,\ 4\ 5\ 6 \\ \hline \square \end{array} \qquad \begin{array}{r} \square\ \ {}^{1}\ \ \ \\ 3,\ 5\ 8\ 4 \\ +\,2,\ 4\ 5\ 6 \\ \hline \square\,\square \end{array} \qquad \begin{array}{r} \square\ {}^{1}\ {}^{1}\ \ \ \\ 3,\ 5\ 8\ 4 \\ +\,2,\ 4\ 5\ 6 \\ \hline 6,\ \square\ 4\ 0 \end{array}$$

2.

$$\begin{array}{r} \square \\ 4\ 6,\ 1\ 7\ 9 \\ +\,3\ 8,\ 6\ 4\ 4 \\ \hline \square \end{array} \qquad \begin{array}{r} \square\ \ {}^{1}\ \ \ \\ 4\ 6,\ 1\ 7\ 9 \\ +\,3\ 8,\ 6\ 4\ 4 \\ \hline \square\ 3 \end{array} \qquad \begin{array}{r} \square\ \ {}^{1}\ {}^{1}\ \ \ \\ 4\ 6,\ 1\ 7\ 9 \\ +\,3\ 8,\ 6\ 4\ 4 \\ \hline \square\,\square,\square\ 2\ 3 \end{array}$$

3.

$$\begin{array}{r} \square\,\square \\ 5\ 3,\ \cancel{7}\ \cancel{8}\ 5 \\ -\,1\ 4,\ 3\ 8\ 1 \\ \hline \square \end{array} \qquad \begin{array}{r} {}^{6}\ {}^{15}\ \ \ \\ 5\ 3,\ \cancel{7}\ \cancel{8}\ 5 \\ -\,1\ 4,\ 3\ 8\ 1 \\ \hline \square\ 7\ 4 \end{array} \qquad \begin{array}{r} \square\,\square\ {}^{6}\ {}^{15}\ \ \ \\ \cancel{5}\,\cancel{3},\ \cancel{7}\ \cancel{8}\ 5 \\ -\,1\ 4,\ 3\ 8\ 1 \\ \hline \square\ 9,\ 3\ 7\ 4 \end{array}$$

4.

$$\begin{array}{r} 8\ 0,\ 0\ 9\ 2 \\ -\,6\ 6,\ 5\ 7\ 1 \\ \hline \square\,\square \end{array}$$

Regroup to make hundreds.

80,000 = 70,000 + ______

10,000 = 9,000 + ______

1,000 = ____ hundreds

$$\begin{array}{r} \square\,\square\ \square\ \ \ \ \ \\ \cancel{8}\ \cancel{0},\ \cancel{0}\ 9\ 2 \\ -\,6\ 6,\ 5\ 7\ 1 \\ \hline \square\,\square,\square\ 2\ 1 \end{array}$$

Think • Pair • Share

MP4 **5.** To understand the problem at the right, Hunter writes the numbers in expanded form. How can Hunter regroup to keep subtracting and finish the problem?

$$\begin{array}{r} 7,034 \\ -5,653 \\ \hline 1 \end{array} \longrightarrow \begin{array}{r} 7,000+\square+30+4 \\ -5,000+600+50+3 \\ \hline 1 \end{array}$$

Independent Practice

Write the missing digits to complete each exercise.

1.
$$\begin{array}{rrrrr} & & \square & & \\ & 7, & 3 & 6 & 0 \\ + & 1, & 4 & 5 & 9 \\ \hline & 8, & \square & 1 & \square \end{array}$$

2.
$$\begin{array}{rrrrrr} & & \square & & \square & \\ & & 8, & 5 & 4 & 8 \\ + & & 8, & 9 & 3 & 4 \\ \hline & \square & \square, & 4 & \square & 2 \end{array}$$

3.
$$\begin{array}{rrrrrr} & & \square & 1 & \square & \\ & 5 & 2, & 7 & 2 & 6 \\ + & 4 & 4, & 9 & 7 & 6 \\ \hline & \square & \square, & \square & \square & 2 \end{array}$$

4.
$$\begin{array}{rrrrr} & & \square & 13 & \\ & 6, & \cancel{7} & \cancel{3} & 9 \\ - & 2, & 6 & 8 & 3 \\ \hline & 4, & 0 & \square & \square \end{array}$$

5.
$$\begin{array}{rrrrr} & & & \square & \square \\ & 9, & 2 & \cancel{9} & \cancel{7} \\ - & 2, & 1 & 4 & 8 \\ \hline & 7, & \square & 4 & \square \end{array}$$

6.
$$\begin{array}{rrrrrr} & \square & \square & 4 & \square & 10 \\ & \cancel{4} & \cancel{1}, & \cancel{5} & \cancel{2} & 0 \\ - & 1 & 5, & 4 & 6 & 1 \\ \hline & 2 & \square, & 0 & \square & 9 \end{array}$$

Add or subtract to check your answers.

7.
$$\begin{array}{rrrrr} & & & \square & \\ & 7, & 1 & 4 & 9 \\ + & 2, & 1 & 4 & 8 \\ \hline & 9, & 2 & \square & \square \end{array}$$

8.
$$\begin{array}{rrrrrr} & & \square & \square & \square & \square \\ & \cancel{1} & \cancel{7}, & \cancel{4} & \cancel{8} & \cancel{2} \\ - & & 8, & 9 & 3 & 4 \\ \hline & & \square, & 5 & \square & 8 \end{array}$$

9.
$$\begin{array}{rrrrrr} & & \square & \square & \square & \square \\ & 9 & \cancel{7}, & \cancel{7} & \cancel{0} & \cancel{2} \\ - & 4 & 4, & 9 & 7 & 6 \\ \hline & 5 & \square, & 7 & \square & 6 \end{array}$$

Estimate each sum. Then find the actual sum. Show how you regroup.

10. Estimate: ______________

$$\begin{array}{r} 38{,}434 \\ +\quad 2{,}575 \\ \hline \end{array}$$

11. Estimate: ______________

$$\begin{array}{r} 59{,}149 \\ +52{,}612 \\ \hline \end{array}$$

12. Estimate: ______________

$$\begin{array}{r} 223{,}458 \\ +198{,}837 \\ \hline \end{array}$$

13. Estimate: ______________

$$\begin{array}{r} 464{,}772 \\ +257{,}432 \\ \hline \end{array}$$

Independent Practice

Estimate each difference. Then find the actual difference. Show how you regroup.

14. Estimate: ______

$$\begin{array}{r} 9{,}185 \\ -3{,}964 \\ \hline \end{array}$$

15. Estimate: ______

$$\begin{array}{r} 17{,}268 \\ -12{,}970 \\ \hline \end{array}$$

16. Estimate: ______

$$\begin{array}{r} 640{,}593 \\ -286{,}406 \\ \hline \end{array}$$

17. Estimate: ______

$$\begin{array}{r} 500{,}642 \\ -371{,}351 \\ \hline \end{array}$$

For problems 18–19, use the table at the right.

Greatest Known Ocean Depths

Ocean	Depth in feet
Pacific	36,198
Atlantic	30,246
Southern	23,736
Arctic	18,456

MP1 **18.** Naomi says the combined depth of the Southern Ocean and Artic Ocean is less than the depth of the Pacific Ocean. Estimate and then find the combined depth. Is Naomi correct?

Answer ______

MP2 **19.** Vince wants to know how much deeper the Atlantic Ocean is than the Southern Ocean. What is the difference between the greatest depths of the two oceans?

Answer ______

MP7 **20.** Naomi rounds the greatest depth of the Pacific Ocean to 36,200 feet. The deepest a scuba diver has gone underwater is 1,044 feet. What is the difference between the two depths?

Answer ______

Independent Practice

MP7 **21.** Michael made a mistake while subtracting. What mistake did Michael make?

$$\begin{array}{r} 5,5\,4\,3 \\ -4,8\,9\,5 \\ \hline 1,3\,5\,2 \end{array}$$

MP6 **22.** Rosie says that when you regroup while adding, the amount that you add to the next place to the left will always be 1. Is Rosie correct? Explain.

For problems 23–26, use the table at the right. Use addition or subtraction to check your answer.

Aquarium Visitors

Month	Number of Visitors
May	85,628
June	97,134
July	101,942
August	123,291
September	64,417

MP1 **23.** How many people visited the aquarium in May and June?

Show your work.

Answer ____________________

MP1 **24.** How many visitors went to the aquarium in June, July, and August?

Show your work.

Answer ____________________

Independent Practice

MP1 **25.** How many more visitors went to the aquarium in August than in June?

Show your work.

Answer ____________________

MP2 **26.** What is the difference between the greatest and least numbers of visitors to the aquarium in one month?

Show your work.

Answer ____________________

MP6 **27.** Trey is teaching his brother how to add.
Here is how his brother added.
Is Trey's brother's work correct?
If not, what mistake did Trey's brother make?

$$\begin{array}{r} \scriptstyle 1\;1\;1\;1\;1\;1 \\ 123{,}456 \\ +\,123{,}456 \\ \hline 357{,}913 \end{array}$$

Answer ____________________

Justify your answer using words, drawings, or numbers.

MP3 **28.** Trey says that before you subtract, you can regroup all of the values that you need to at once. Does this method give you the same difference as when you regroup one place at a time, while you subtract?

Answer ____________________

Justify your answer using words, drawings, or numbers.

Lesson

10 Multiply Whole Numbers: Use Place Value

Essential Question:
How can you use place value and patterns to multiply?
4.NBT.5

Words to Know:
partial products

Guided Instruction

In this lesson you will learn how to use place value to multiply greater factors.

Understand: Products of tens, hundreds, and thousands

At a swimming pool guests are charged $5 to swim for the day. How much money does the pool receive if 20, 200, or 2,000 people swim?

To find each product, use the fact 5 × 2 and place-value patterns.

Remember!
Each place in a number is 10 times the value of the place to its right.

5 × 20
= 5 × 2 tens
= 10 tens
= 100

5 × 200
= 5 × 2 hundreds
= 10 hundreds
= 1,000

5 × 2,000
= 5 × 2 thousands
= 10 thousands
= 10,000

The pool receives $100 for 20 people, $1,000 for 200 people, and $10,000 for 2,000 people.

You can also multiply the nonzero digits, 5 × 2 = 10 and then compare the number of zeros in the factors with the number of zeros after the 10 in each product. You can use this pattern with zeros to multiply.

5 × 20 = 100
5 × 200 = 1,000
5 × 2,000 = 10,000

Note that it works in other problems. Look at the problem 10 × 20. There are two zeros in the factors.
Multiply the nonzero digits: 1 × 2 = 2.
Then write the number of zeros in the factors to the end of that product, to show the correct place value.

$1\underline{0} \times 2\underline{0} = 1 \text{ ten} \times 2 \text{ tens}$
$= 2 \text{ hundreds}$
$= 2\underline{00}$

This pattern helps you when you multiply tens, hundreds, and thousands.

Guided Instruction

Understand: Place value and partial products

Rob is using a garden hose to fill a swimming pool. In one hour there will be 924 gallons of water. How many gallons of water will the pool have in it after 6 hours?

To find the total number of gallons, you can multiply 6 times the value of each digit in 924. Partial products are formed by multiplying the value of each digit by a factor.

Find the partial products.

Multiply the ones first.
Multiply 6 times the value of the digit 4.
Write the product below.

$$\begin{array}{r} 924 \\ \times\quad 6 \\ \hline 24 \end{array} \leftarrow 6 \times 4 \text{ ones}$$

Next, multiply 6 times the value of the digit 2.

$$\begin{array}{rl} 924 & \\ \times\quad 6 & \\ \hline 24 & \leftarrow 6 \times 4 \text{ ones} \\ 120 & \leftarrow 6 \times 2 \text{ tens} \end{array}$$

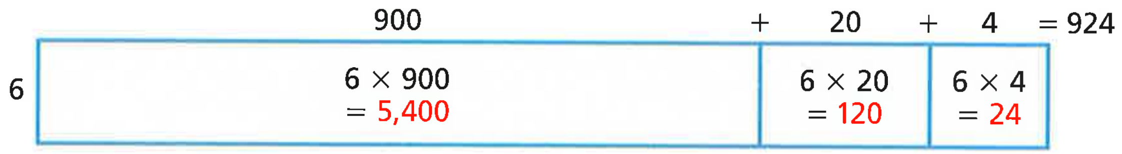
Then multiply 6 times the value of the digit 9.

$$\begin{array}{rl} 924 & \\ \times\quad 6 & \\ \hline 24 & \leftarrow 6 \times 4 \text{ ones} \\ 120 & \leftarrow 6 \times 2 \text{ tens} \\ 5400 & \leftarrow 6 \times 9 \text{ hundreds} \end{array}$$

Now add all the partial products.
924 × 6 = 24 + 120 + 5,400 = 5,544

$$\begin{array}{r} 924 \\ \times\quad 6 \\ \hline 24 \\ 120 \\ +\,5400 \\ \hline 5,544 \end{array}$$

You can use an area model to show the partial products in the final product. Use the value of the digits in each factor as the side lengths of the area model.



The pool will have 5,544 gallons of water in it after 6 hours.

Lesson 10 Multiply Whole Numbers: Use Place Value

Guided Instruction

Connect: What you know about multiplication and partial products

A lifeguard works at a pool for 28 hours each week. How many hours will the lifeguard work in 12 weeks?

To multiply two 2-digit numbers, use the expanded form of each factor to find partial products. Begin by estimating to judge the reasonableness of your exact answer.

Step 1

Estimate the product. You can round down 12 and round up 28.
$12 \times 28 \rightarrow 10 \times 30 = 300$.

Find the products of the nonzero factors. Count the zeros in the factors. Write the total number of zeros at the end of the product.

Step 2

Make an area model to show the partial products you need to find. In the area model the sides represent the 12 weeks, 12 = 10 + 2, and the 28 hours, 28 = 20 + 8, worked each week.

The sum of the areas of each part is the total product. Side lengths of the rectangle:
12 = 10 + 2   28 = 20 + 8

	20	+ 8
10	10×20 $= 200$	10×8 $= 80$
+ 2	2×20 $= 40$	2×8 $= 16$

Step 3

Now multiply to find each partial product.

$$\begin{array}{rl} 28 & \\ \times\,12 & \\ \hline 16 & \leftarrow 2 \times 8 \end{array} \qquad \begin{array}{rl} 28 & \\ \times\,12 & \\ \hline 16 & \leftarrow 2 \times 8 \\ 40 & \leftarrow 2 \times 20 \end{array} \qquad \begin{array}{rl} 28 & \\ \times\,12 & \\ \hline 16 & \leftarrow 2 \times 8 \\ 40 & \leftarrow 2 \times 20 \\ 80 & \leftarrow 10 \times 8 \end{array} \qquad \begin{array}{rl} 28 & \\ \times\,12 & \\ \hline 16 & \leftarrow 2 \times 8 \\ 40 & \leftarrow 2 \times 20 \\ 80 & \leftarrow 10 \times 8 \\ 200 & \leftarrow 10 \times 20 \end{array}$$

Step 4

Add the partial products to find the total product. How does this compare to your estimate?

$$\begin{array}{r} 28 \\ \times\,12 \\ \hline 16 \\ 40 \\ 80 \\ +\,200 \\ \hline 336 \end{array}$$

The exact answer, 336, is reasonably close to the estimate of 300.

➡ The lifeguard will work 336 hours in 12 weeks.

Guided Practice

Use place value to find each product.

1. 4 × 700
   = 4 × ____ hundreds
   = ____ hundreds
   = 2,800

2. 9 × 5,000
   = 9 × ____ thousands
   = ____ thousands
   = ________

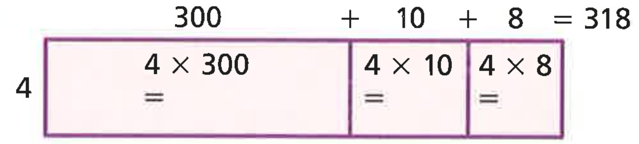
Complete the multiplication pattern.

3. 8 × 3 = ____
   8 × 30 = 240
   8 × 300 = ________
   8 × 3,000 = ________

4. 5 × 6 = ____
   5 × 60 = ________
   5 × ________ = 3,000
   5 × 6,000 = ________

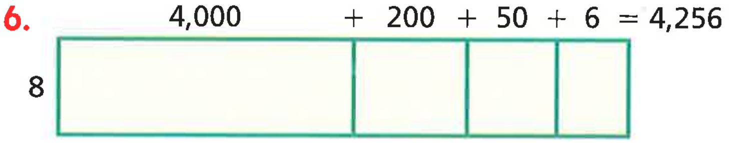
Use the area model to multiply. Label the model with the partial products. Then complete the multiplication.

5.



```
  318
×   4
_____
        ← 4 × 8 ones
        ← 4 × 1 ten
+ ____  ← 4 × 3 hundreds
```

6.



```
  4,256
×     8
_______

+ _____
```

Think • Pair • Share

MP2 7. Mia says the product of 21 and 53 is about the same as 2 × 500. Is Mia correct? Explain her method for estimating, and why it does or does not work.

Independent Practice

Use multiplication patterns to multiply.

1. $200 \times 4 =$ ____ ____ ____

2. $3{,}000 \times 4 =$ ____ ____ ____ ____ ____

3. $20 \times 30 =$ ____ ____ ____

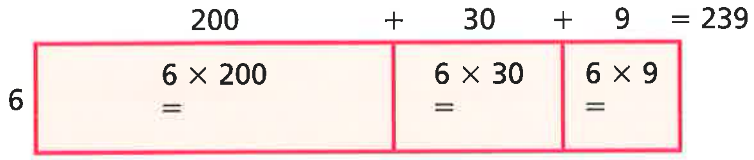
4. $9 \times 500 =$ ________

5. $8 \times 7{,}000 =$ ________

6. $50 \times 40 =$ ________

Use the area model to multiply. Label the model with the partial products. Then complete the multiplication.

7.

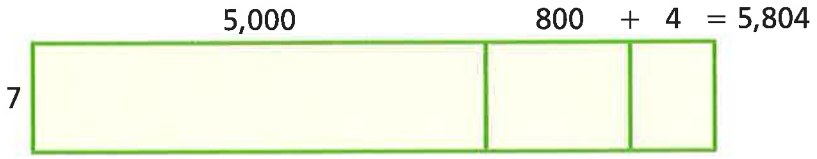


$6 \times 239 =$ ________ + ________ + ____

$=$ ________

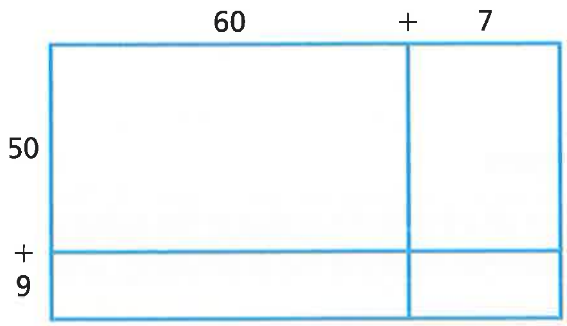
8.

$$\begin{array}{r} 5{,}804 \\ \times \quad 7 \\ \hline \\ \\ + \quad \\ \hline \end{array}$$



9.

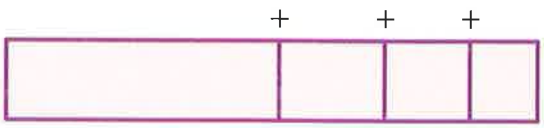
$$\begin{array}{r} 67 \\ \times \ 59 \\ \hline \\ \\ + \quad \\ \hline \end{array}$$



Independent Practice

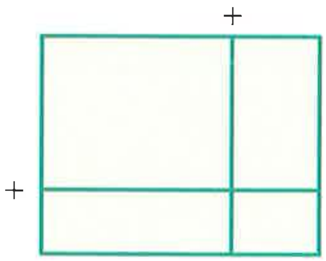
Multiply. Use the area model to help you.

10. A store at the beach sells 9,586 bottles of sunscreen in one summer. Each bottle costs $8. How much does the store receive for sunscreen?



9,586 × $8 = ______________________

11. Crystal is training for a swimming competition. She swims 72 feet in 1 minute. If Crystal could keep up this pace, how far would she swim in 85 minutes?



72 × 85 = ______________________

Solve the problem.

12. Oscar uses the expanded form of the factors to find and record partial products without a model. Use Oscar's method to multiply 8,954 by 9.

$$\begin{array}{r} 8{,}000 + 900 + 50 + 4 \\ \times \qquad\qquad\qquad\qquad 9 \\ \hline \boxed{} \\ \boxed{8{,}1} \\ \boxed{} \\ +\qquad 36 \\ \hline \boxed{} \end{array} \rightarrow \begin{array}{r} 8{,}954 \\ \times \quad 9 \\ \hline \boxed{} \end{array}$$

Estimate each product. Then multiply.

13. 3,362 × 5

14. 77 × 43

Independent Practice

MP2 **15.** George multiplies 3 times 3,519. His work is shown to the right. What mistake does George make?

$$\begin{array}{r} 3{,}519 \\ \times \quad\;\; 3 \\ \hline 27 \\ 30 \\ 150 \\ +9{,}000 \\ \hline 9{,}207 \end{array}$$

MP7 **16.** Robin is multiplying 80 times 42. How many partial products will Robin have to add? Explain your answer.

Solve the problems.

MP1 **17.** An airline has planes in two sizes. The small planes have 135 seats, and the larger planes have 172 seats. What is the total number of seats on 5 small planes and 4 large planes?

Show your work.

Answer ______________________________

MP2 **18.** A large, older airplane uses 1,193 gallons of fuel each hour that it is flying. How many gallons of fuel will the plane use for a 6-hour flight?

Show your work.

Answer ______________________________

Independent Practice

MP2 **19.** Chicago is about 4,254 miles from Honolulu. If an airplane could make 3 trips between these cities in one day, how many miles would the airplane fly altogether?

Show your work.

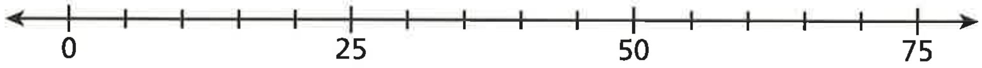
Answer ______________________

MP1 **20.** A group of 55 tourists order a special meal at a restaurant. The cost of each person's meal is $38. Each person will pay a $7 tip. What is the total restaurant bill for the group?

Show your work.

Answer ______________________

MP4 **21.** Jessica uses a number line to find the product 4 × 15. She skip-counts by 15 four times, and reaches a product of 60.



Miles says this is not the best method for multiplying greater numbers. Do you agree with Miles? Give one example.

Answer ______________________

Justify your answer using words, drawings, or numbers.

Lesson

11 Multiply Whole Numbers: Use Properties of Operations

Essential Question:
How can you break apart greater factors to multiply?
4.NBT.5

Guided Instruction

In this lesson you will learn how to multiply greater whole numbers using the Distributive and other properties.

Understand: The Distributive Property and expanded form

To raise money, a school will have a concert and it will charge $5 for each ticket to the concert. How much will the school receive if the school sells 384 tickets?

To find how much the school will receive you can use the Distributive Property to multiply 5 × 384.

First, decompose 384, the greater factor.
Write 384 in expanded form, to break it into a sum of hundreds, tens, and ones.

$5 \times 384 = 5 \times (300 + 80 + 4)$

Multiply each addend by 5, the lesser factor.
Then add the partial products.

$$\begin{aligned} 5 \times (300 + 80 + 4) &= (5 \times 300) + (5 \times 80) + (5 \times 4) \\ &= 1{,}500 + 400 + 20 \\ &= 1{,}920 \end{aligned}$$

You use the Distributive Property when you multiply the addends in the expanded form of one factor by the other factor.

The school will receive $1,920 from the sale of 384 tickets.

Use an area model to represent the partial products.

	300 +	80 +	4 = 384
5	5 × 300 = 1,500	5 × 80 = 400	5 × 4 = 20

The area model can be helpful when using the Distributive Property. It shows how multiplying 5 times the hundreds, tens, and ones in 384 is the same as multiplying 5 × 384.

What are some other ways you can break apart 384?

Guided Instruction

Understand: The Distributive Property and two-digit factors

There are 64 fourth graders at the school. If each fourth grader sells 15 concert tickets, how many tickets will the fourth graders sell altogether?

To find how many tickets will be sold, multiply 64 × 15.

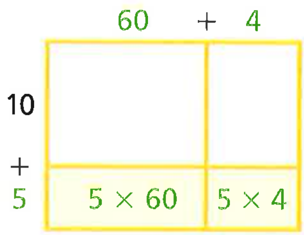
Write the multiplication using the expanded form of both factors.
Use the Commutative Property to change the order of the factors.
$15 \times 64 = (10 + 5) \times (60 + 4)$

You can use an area model to help you. Use the value of the digits in each factor as the side lengths of the area model.

You can use the Distributive Property more than once. To distribute, multiply *each addend* for 64 by *each addend* for 15.

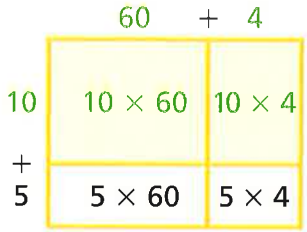
First, multiply by 5, the ones digit in 15.

$$5 \times (60 + 4) = (5 \times 60) + (5 \times 4)$$



Next, multiply by 10, the tens digit in 15.

$$10 \times (60 + 4) = (10 \times 60) + (10 \times 4)$$



Then, write an equation showing the sum of the four partial products and solve.

$$\begin{aligned}(10 + 5) \times (60 + 4) &= (10 \times 60) + (10 \times 4) + (5 \times 60) + (5 \times 4)\\ &= 600 + 40 + 300 + 20\\ &= (600 + 300) + (40 + 20)\\ &= 900 + 60\\ &= 960\end{aligned}$$

Remember! You can use the Commutative and Associative properties to help you add.

The fourth graders will sell 960 concert tickets altogether.

Look at the highlighted digits to the right. How are they related to the partial products that you found above?

$$\begin{array}{r} 64 \\ \times 15 \\ \hline \end{array} \qquad \begin{array}{r} 64 \\ \times 15 \\ \hline \end{array}$$

Guided Instruction

Connect: What you know about the Distributive Property and other multiplication strategies

Ruby and Omar both multiply 1,296 × 3. Ruby uses the expanded form of 1,296 and the Distributive Property. Omar multiplies a number close to 1,296 and uses other properties. Can both Ruby and Omar find the correct product using their different methods?

Try each method to compare Ruby and Omar's work.

Step 1

Ruby's Method

Write the problem using the expanded form of 1,296.	$1,296 \times 3$ $= (1,000 + 200 + 90 + 6) \times 3$
Multiply each addend for 1,296 by 3.	$= (1,000 \times 3) + (200 \times 3) + (90 \times 3) + (6 \times 3)$
Multiply inside the parentheses.	$= 3,000 + 600 + 270 + 18$
Add the partial products.	$= 3,888$

Step 2

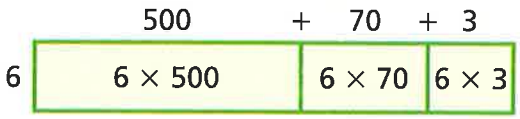
Omar's Method

Omar uses an estimate. 1,296 is almost 1,300, or 13 hundreds.	$1,300 \times 3 = 13 \times 100 \times 3$
He uses the Commutative Property to reorder the factors.	$= 13 \times 3 \times 100$
Omar uses the Associative Property to group 13 and 3. Then he uses multiplication patterns with zero to multiply.	$= (13 \times 3) \times 100$ $= 39 \times 100$ $= 3,900$
Since 1,300 − 1,296 = 4, he compensates for the difference. There are 3 groups of 1,300 in 3,900, so he subtracts 4 three times.	$1,296 \times 3 = 3,900 - (4 \times 3)$ $= 3,900 - 12$ $= 3,888$

➡ Yes, Ruby and Omar can both find the correct product using their different methods.

Guided Practice

Use the Distributive Property to multiply. You can use the area model to help.



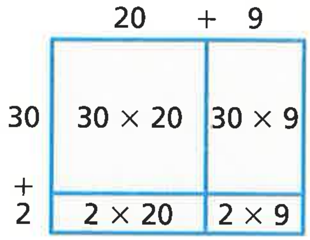
1. 6 × 573

= 6 × (____ + 70 + ____)

= (____ × ____) + (6 × 70) + (____ × ____)

= ________ + 420 + ____

= ________



2. 32 × 29

= (____ + 2) × (____ + 9)

= (30 × ____) + (30 × ____) + (2 × ____) + (2 × ____)

= ____ + 270 + 40 + ____

= ____

Use the Distributive Property and expanded form to multiply.

3. 4 × 6,725 = ____ × (6,000 + ____ + ____ + 5)

= (4 × 6,000) + (________) + (________) + (4 × 5)

= ________ + 2,800 + ____ + ____

= ________

Think • Pair • Share

MP6 **4.** Leo multiplies 4 × 285 as shown below.

4 × (250 + 30 + 5) = 1,000 + 120 + 20 = 1,140

Does Leo use the Distributive Property? Does he find the correct product? Explain why or why not.

Independent Practice

Use the Distributive Property and expanded form to multiply.

1. 5 × 468 = 5 × (____ + ____ + 8)

= (5 × 400) + (__________) + (__________)

= __________ + 300 + ____

= __________

2. 7 × 5,619 = 7 × (__________ + ____ + 10 + ____)

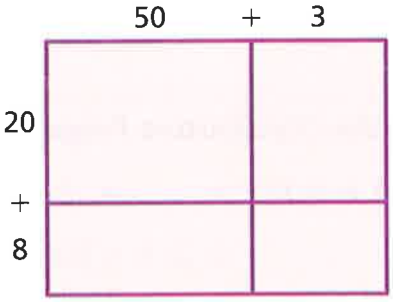
= (__________) + (__________) + (7 × 10) + (7 × 9)

= __________ + __________ + ____ + ____

= __________

Use the Distributive Property and area model to multiply. Label the model with the partial products to help you.

3. 28 × 53



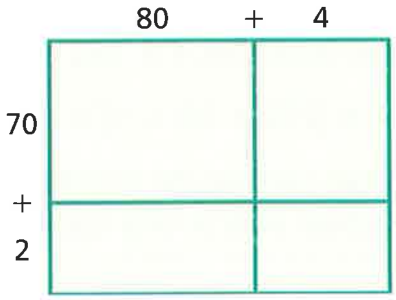
28 × 53 = (____ + 8) × (____ + 3)

= (20 × ____) + (____ × 3) + (8 × ____) + (____ × 3)

= __________ + 60 + ____ + 24

= __________

4. 72 × 84



72 × 84 = (__________) × (__________)

= (__________) + (__________) + (__________) + (__________)

= ________ + ________ + ________ + ________

= __________

Independent Practice

Circle the correct answer.

5. Each equation below shows a different method for multiplying 40 × 48. Which equation is incorrect?

a. $40 \times 48 = 40 \times (40 + 8)$

b. $40 \times 48 = (20 + 20) \times (40 + 8)$

c. $40 \times 48 = 10 \times 4 \times (40 + 8)$

d. $40 \times 48 = (2 \times 20) + (40 + 8)$

Justify your answer using words, drawings, or numbers.

Match each equation with the correct product.

6. $8 \times 198 = 8 \times (100 + 90 + 8)$ ____

7. $3 \times 325 = 3 \times (300 + 20 + 5)$ ____

8. $5 \times 264 = 5 \times (200 + 60 + 4)$ ____

9. $16 \times 75 = (10 + 6) \times (70 + 5)$ ____

10. $27 \times 36 = (20 + 7) \times (30 + 6)$ ____

Products

a. 972

b. 975

c. 1,200

d. 1,320

e. 1,584

Estimate the product. Then multiply using the properties of operations. Show your work.

11. 4,462 × 9

Estimate: ____________________

Answer ____________________

12. 34 × 56

Estimate: ____________________

Answer ____________________

Independent Practice

MP6 **13.** Compare your work for exercises 11 and 12. How is multiplying by a factor with one digit different from multiplying two factors with two digits?

MP2 **14.** Jenny multiplies 9,059 × 6 as shown below.

$$\begin{array}{rr}
 & 9,059 \\
 & \times \quad\quad 6 \\
\cline{2-2}
6 \times 9,000 & 54,000 \\
6 \times 0 & 0 \\
6 \times 50 & 300 \\
6 \times 9 & + \quad\quad 54 \\
\cline{2-2}
 & 54,354
\end{array}$$

How is Jenny using the Distributive Property with this method? Explain.

Solve the problems using the method of your choice.

MP4 **15.** The students are helping to set up chairs for their school's concert. They make 24 rows with 32 chairs in each row. How many chairs do the students set up altogether?

Show your work.

Answer ______________________________

MP2 **16.** A grocery store uses 3,452 plastic bags in one week. If the store uses 5 times as many bags for the whole month, how many bags does the store use that month?

Show your work.

Answer ______________________________

Independent Practice

MP1 **17.** An athlete training for the Olympics eats 9,655 calories of food each day for 7 days in a row. What is the total number of calories the athlete eats that week?

Show your work.

Answer ____________________

MP4 **18.** The soccer field at the park is 48 yards wide and 75 yards long. There are 36 inches in 1 yard. What are the width and length of the field in inches?

Show your work.

Answer ____________________

MP8 **19.** Brian says that the product of two whole numbers with 2 digits each will always have 4 digits. Do you agree? Give at least one example to support your answer.

Answer ____________________

Justify your answer using words, drawings, or numbers.

Lesson
12 Divide Whole Numbers: Use Place Value

Essential Question:
How can place value and estimation help you divide?
4.NBT.6

Words to Know:
- **dividend**
- **divisor**
- **quotient**
- **compatible numbers**

Guided Instruction

In this lesson you will learn how to use place value to estimate quotients and divide greater whole numbers.

Understand: Quotients of tens, hundreds, and thousands

> An article in a magazine states that scientists estimate that lightning strikes about 200 times around the world every 5 seconds. Following this pattern, find the number of times lightning strikes each second.

To find the number of lightning strikes in 1 second, divide 200 by 5.

In this division 200 represents the dividend, the number you divide; 5 represents the divisor, the number you divide by in the division to find a quotient, the unknown number that is the answer to the problem.

$$\underset{\text{dividend}}{200} \div \underset{\text{divisor}}{5} = \underset{\text{quotient}}{n}$$

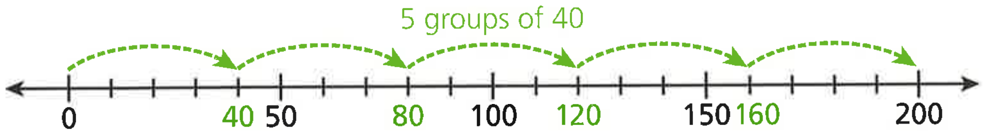
Use the fact $20 \div 5 = 4$ and the pattern of zeros to find the quotient, n.

$20 \div 5 = 4$ $\qquad$ $200 \div 5 = n$

$20 \text{ tens} \div 5 = 4 \text{ tens} \longrightarrow 200 \div 5 = 40$, so $n = 40$

Lightning strikes about 40 times each second.

You can use a number line to show a model of $200 \div 5 = 40$.



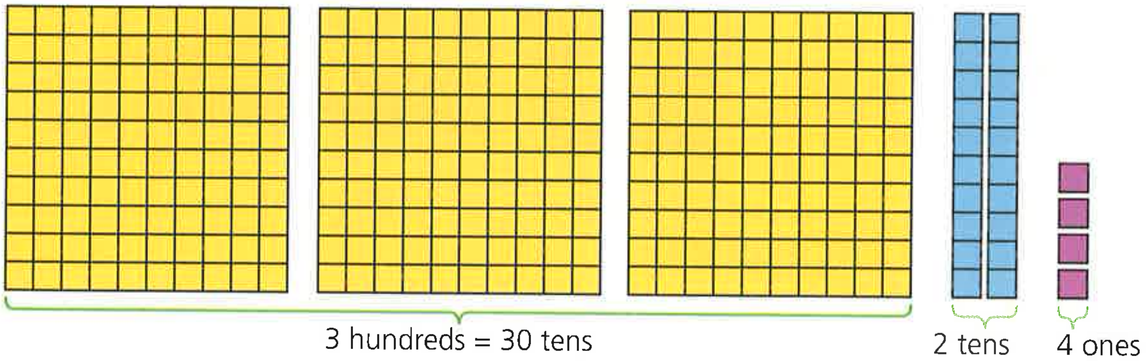
Continue the pattern of zeros to find $2{,}000 \div 5$. How can you change the number line to model $2{,}000 \div 5$?

Guided Instruction

Understand: Place value and regrouping

Julia will drive 324 miles to go camping. If Julia drives the same number of miles each hour, how many miles should Julia drive each hour to reach the camp in 6 hours?

To find the number of miles that Julia has to drive each hour, divide 324 by 6. Use place value models to represent the miles. Divide 3 hundreds, 2 tens, 4 ones into 6 equal groups.



To find 324 ÷ 6, find the partial quotients. Then add them.
Look at the model. Regroup 3 hundreds as 30 tens.
30 tens + 2 tens = 32 tens total to share equally in groups.

Divide 32 tens into 6 equal groups.
32 tens ÷ 6 = 5 tens in each group
and 2 tens left over.

Partial Quotients
4 ones = 4
5 tens = 50

$$\begin{array}{r} 4 \\ 50 \\ 6\overline{)324} \\ -\underline{300} \\ 24 \\ -\underline{24} \\ 0 \end{array}$$

no remainder

2 tens left over is not enough to make 6 groups of ten.

Regroup the 2 left over tens as 20 ones.
20 ones + 4 ones = 24 ones to share

Divide the 24 ones into 6 equal groups.
24 ones ÷ 6 = 4 ones in each group.

Add the partial quotients.
324 ÷ 6 = 50 + 4 = 54

Julia should drive 54 miles each hour to reach the camp in 6 hours.

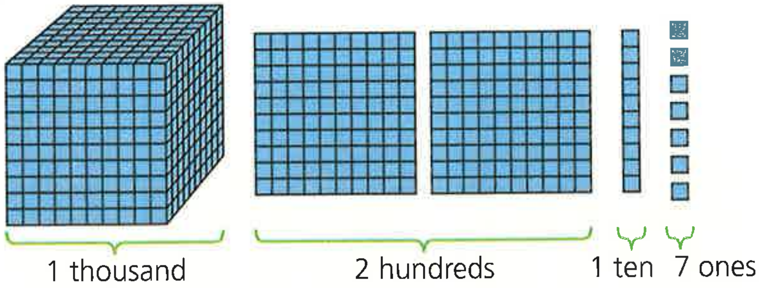
Lesson 12 Divide Whole Numbers: Use Place Value

Guided Instruction

Connect: What you know about regrouping to divide whole numbers into equal groups

A television manufacturer puts 4 batteries in each remote controller. The manufacturer has 1,217 batteries. How many remote controllers can the manufacturer fill with batteries?

To find the number of remote controllers, divide 1 thousand, 2 hundreds, 1 ten, 7 ones by 4. 1,217 ÷ 4



Step 1

Regroup 1 thousand as 10 hundreds.
10 hundreds + 2 hundreds
= 12 hundreds
Divide 12 hundreds by 4.
12 hundreds ÷ 4 = 3 hundred groups of 4.

Step 2

There are no hundreds left to divide.
Regroup 1 ten as 10 ones.
10 ones + 7 ones = 17 ones; Divide 17 ones by 4.
17 ones ÷ 4 = 4 groups of 4 and 1 one left over.

Partial Quotients

$$\begin{array}{rl} \left.\begin{array}{r} 4 \\ 300 \end{array}\right\} & \begin{array}{l} 4 \text{ ones} = 4 \\ 3 \text{ hundreds} = 300 \end{array} \\ \hline 4\overline{)1,217} & \\ -\underline{1,200} & \\ 17 & \\ -\underline{16} & \\ 1 & \leftarrow \text{Remainder} \end{array}$$

Step 3

Add the partial quotients.
1,217 ÷ 4 = 304 groups of 4 batteries and 1 battery left over.

The 1 battery left over does not make another group of 4.

1217 ÷ 4 = 304 R1

Remember!
A remainder (R) is the amount left over after dividing.

Step 4

You can estimate to make sure that the answer is reasonable.
Use a number that is close to 1,217 and easy to divide by 4.

Estimate 1,200 ÷ 4 = 300; about 300 groups
Answer 1,217 ÷ 4 = 304 R1

1,200 and 4 are compatible numbers. Compatible numbers are those that are easy to compute mentally.

The estimate 300 is close to 304 R1, so 304 R1 is a reasonable answer.

➡ The manufacturer can fill 304 remote controllers with 1,216 batteries.

Guided Practice

Complete exercises 1 and 2 to solve the problem. Show your work.

Arly's Movie Triplex has a total of 825 seats in 3 theaters that are the same size. How many seats are in each theater?

1. Estimate the number of seats in each theater. Use compatible numbers.

Estimate 825 ÷ 3. ____________

2. Divide 825 to find the actual quotient.
825 = 8 hundreds + 2 tens + 5 ones

$$\begin{array}{r} 5 \\ 70 \\ 200 \\ 3\overline{)825} \\ -\,600 \\ \hline 225 \\ -\,210 \\ \hline 15 \\ -\,15 \\ \hline 0 \end{array} \quad \left.\begin{array}{r} 5 \\ 70 \\ 200 \end{array}\right\} 275$$

a. Divide the hundreds. 8 hundreds ÷ 3

There are 3 groups of ____ hundreds in 825.

____ hundreds represent seats shared equally among the 3 theaters. ____ hundreds are left over.

b. Regroup the hundreds that are left over.

2 hundreds = ____ tens

____ tens + 2 tens = ____ tens

c. Divide the tens.

There are 3 groups of ____ tens in 22 tens.

____ tens are shared equally among the 3 theaters.

____ ten is left over.

d. Regroup the 1 ten left over with the 5 ones.

1 ten = ____ ones

____ ones + ____ ones = ____ ones to share equally among the 3 theaters.

e. 15 ones = 3 groups of ____ ones.
There are 275 seats in each theater.

Think • Pair • Share

MP7 **3.** Is the actual quotient close to the estimated quotient? Explain why it is helpful to estimate before solving the problem.

Independent Practice

Use division patterns with zeros to find each quotient.

1. 500 ÷ 5 =

____ ____ ____

2. 6,000 ÷ 3 =

____ ____ ____ ____

3. 200 ÷ 4 =

____ ____

4. 2,400 ÷ 8 =

____

5. 1,800 ÷ 9 =

____

6. 4,000 ÷ 5 =

____

Find each quotient. You can use a model to help.

7. 620 ÷ 5 =

____

8. 337 ÷ 9 =

____ R ____

9. 2,908 ÷ 4 =

____

10. 2,909 ÷ 4 =

__________ R ____

11. 197 ÷ 8 =

____ R ____

12. 508 ÷ 5 =

__________ R ____

13. 1,344 ÷ 6 =

____

14. 7,249 ÷ 8 =

__________ R ____

15. 5,320 ÷ 7 =

____

16. 4,545 ÷ 3 =

____

Independent Practice

Estimate each quotient. Use compatible numbers.

17. 71 ÷ 3

Estimate: ____________________

18. 409 ÷ 8

Estimate: ____________________

19. 3,514 ÷ 9

Estimate: ____________________

20. 5,247 ÷ 5

Estimate: ____________________

Divide. Then estimate to find if your quotient is reasonable. Show your work.

21. 421 ÷ 3 = ____________

Estimate: ____________________

____

Is your quotient reasonable?

____

22. 4,298 ÷ 7 ____________

Estimate: ____________________

____

Is your quotient reasonable?

____

Circle the correct answer.

23. Which quotient below has a remainder?

a. 248 ÷ 9

b. 700 ÷ 4

c. 2,905 ÷ 5

d. 6,332 ÷ 4

Independent Practice

MP6 **24.** A music store is selling all of its CDs for $9 each. Jordan has a gift card worth $110. He divides $110 by 9 correctly, and says that he can buy 12 R2 CDs with the gift card. Why is Jordan's statement incorrect?

MP5 **25.** Lucy divides 7,062 by 8 as shown at the right. The correct quotient is 882 R6. What mistakes might Lucy have made? Explain at least two mistakes.

7,062 ÷ 8 = 95 R2

Solve the problems.

MP2 **26.** A garden has 4 rain barrels. When the 4 barrels are full, they hold 255 gallons of rain water altogether. If all 4 barrels are the same size, how many gallons of rain water does each barrel hold?

Show your work.

Answer ______________________________

MP1 **27.** A farm offers horseback trail rides on weekends. The riding trail is 3 miles long. If the horses covered 1,221 miles altogether in one month, how many trail rides were taken at the farm?

Show your work.

Answer ______________________________

Independent Practice

MP2 **28.** A statue at an art museum is 8 times as old as a famous painting. If the statue is 2,376 years old, how old is the painting?

Show your work.

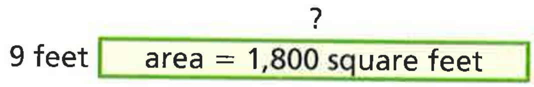
Answer ______________________

MP1 **29.** The Water Warehouse store sells 6,093 bottles of water on Friday. The bottles come in packs of 9 bottles each. On Monday, the store sells 215 fewer packs of water than it sells on Friday. How many packs of water does the store sell on Monday?

Show your work.

Answer ______________________

MP2 **30.** Carmen knows that when you use an area model, you multiply the side lengths to find the area. She says that if the area of the rectangle is 1,800 square feet, then its length must be 200 feet. Is Carmen correct?



Answer ______________________

Justify your answer using words, drawings, or numbers.

MP6 **31.** The bowling league allows no more than 5 people on a team. Is it possible to have 28 teams if there are 142 people?

Answer ______________________

Justify your answer using words, drawings, or numbers.

Lesson

13 Divide Whole Numbers: Use Properties of Operations

Essential Question:
How can breaking apart whole numbers make them easier to divide?
4.NBT.6

Guided Instruction

In this lesson you will learn how to divide using properties and the relationship between multiplication and division.

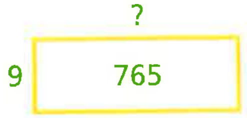
Understand: Division and multiples of the divisor

There are 9 innings in a baseball game. Last season Tom pitched all complete games. If he pitched 765 innings, how many games did Tom pitch last season?

Remember!
Use basic facts and patterns with 0 to find the products.

To find how many games Tom pitched, divide the number of games he pitched by the number of innings in a game. You can use an area model to represent 765 divided by 9. In the area model at the right, the quotient 765 ÷ 9 is the unknown side length.

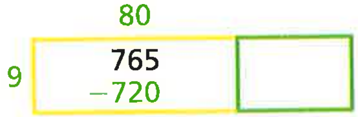
Find a multiple of 9 that is close to but not greater than 765.



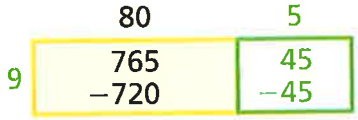
9 × 100 = 900   9 × 90 = 810   9 × 80 = 720

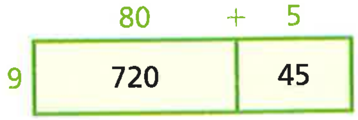
720 is easy to divide: 720 ÷ 9 = 80.

80 is part of the unknown side length of the area model.
Show 720 as the area of the 9 × 80 region.
Subtract this area from 765:
765 − 720 = 45.
Draw another region to show 45 as the remaining area.
Divide by 9 to find the other part of the unknown side length.



45 is a multiple of 9. Divide to find the side length of the new region: 45 ÷ 9 = 5.
Subtract: 45 − 45 = 0. There is nothing left over.
Look at the completed model of 765 ÷ 9 below.





To find the quotient, add the side lengths you found.
80 + 5 = 85

Tom pitched 85 games last season.

What happens if you break 765 into smaller multiples of 9?

Understand: Division and the Distributive Property

A new high-speed train in Japan will travel 2,280 feet in 5 seconds.
How many feet will the high-speed train travel in 1 second?

You can break apart a dividend to divide.
The Distributive Property states that to divide a sum by a number, you can divide each addend of the sum by that number and add the partial quotients.

Remember!
The dividend is the number you divide.

To find how many feet the train will travel in 1 second, divide 2,280 by 5.
Break apart 2,280.

Choose a hundreds number that when multiplied by 5 would be close to 2,280.
$5 \times 400 = 2{,}000$

Subtract.

Next choose a tens number that when multiplied by 5 would be close to 280.

$$\begin{array}{r l} 2{,}280 & \\ -2{,}000 & \leftarrow 5 \times 400 \\ \hline 280 & \\ -250 & \leftarrow 5 \times 50 \\ \hline 30 & \\ -30 & \leftarrow 5 \times 6 \\ \hline 0 & \end{array}$$

$5 \times 50 = 250$. Subtract.

Now choose a ones number that when multiplied by 5 would be close to 30.

Remember!
The divisor is the number you divide by in a division problem.

$5 \times 6 = 30$. Subtract.

Write 2,280 as a sum of 2,000 + 250 + 30.

You can distribute the divisor, 5, the same way you distributed a factor when using the Distributive Property of multiplication.

When dividing 2,280, each addend is divided by 5.
Add these partial quotients to find the answer.

$$\begin{aligned} 2{,}280 \div 5 &= (2{,}000 \div 5) + (250 \div 5) + (30 \div 5) \\ &= 400 + 50 + 6 \leftarrow \text{partial quotients} \\ &= 456 \end{aligned}$$

➡ The high-speed train will travel 456 feet in 1 second.

Guided Instruction

Connect: Multiples and partial quotients

Marco uses a pedometer to count the steps he takes. He takes 2,207 steps in 3 hours. If he takes the same number of steps each hour, how many steps does Marco take in one hour?

STEPS:
2,207
TIME: 180 MIN

To find how many steps Marco takes in one hour, divide 2,207 by 3.
Use the Distributive Property to find 2,207 ÷ 3.

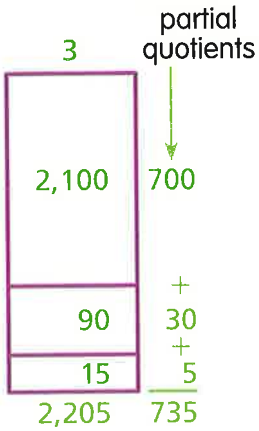
Break apart 2,207. Divide each addend by 3 and add the partial products.
Use basic facts and patterns with 0.

3 × 800 = 2,400   3 × 700 = 2,100   3 × 6 = 1,800

You can use 2,100 as the first addend.
Find the first partial quotient:
2,100 ÷ 3 = 700

Subtract. Find the other partial products. You can record your work either way shown.

$$\begin{array}{r l} 2{,}207 & \\ -\,2{,}100 & \leftarrow 3 \times 700 \\ \hline 107 & \\ -\,90 & \leftarrow 3 \times 30 \\ \hline 17 & \\ -\,15 & \leftarrow 3 \times 5 \\ \hline 2 & \text{remainder} \end{array}$$



Look at the area model.
Add the partial quotients to find 2,205 ÷ 3.

$$\begin{aligned} 2205 \div 3 &= (2{,}100 \div 3) + (90 \div 3) + (15 \div 3) \leftarrow \text{Distributive Property of Division} \\ &= (700 + 30 + 5) \\ &= 735 \end{aligned}$$

Note that the quotient for the full quantity has a remainder of 2.

$2{,}207 \div 3 = 735\ \text{R}2$

$735\ \text{R}2 \longrightarrow (2{,}100 \div 3) + (90 \div 3) + (15 \div 3) + 2$

Since Marco takes 1 extra step for 2 of the 3 hours, increase the quotient by 1 to include the remainder 2 as part of your answer.

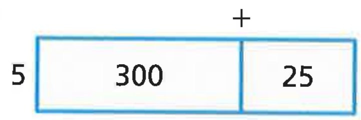
➡ Marco takes about 736 steps in one hour.

- Look at the area model again. Why is the remainder not shown as part of the rectangle?

Guided Practice

Complete each area model. Write the unknown side lengths to find the partial quotients. Then add to find the quotient.

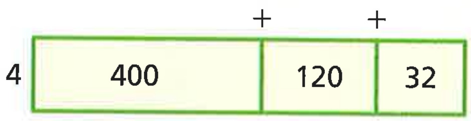
1.



325 ÷ 5 = ____ + ____

= ____

2.

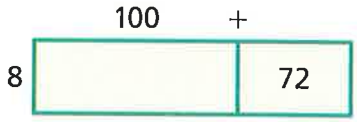


552 ÷ 4 = ____ + ____ + ____

= ____

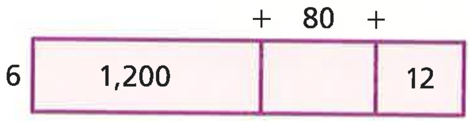
Complete each area model. Then write the quotient below.

3.



872 ÷ 8 = ____

4.



1,692 ÷ 6 = ____

Complete each division.

5. 496 ÷ 2 = (____ ÷ 2) + (80 ÷ 2) + (____ ÷ 2)
= 200 + 40 + 8
= ____

6. 2,189 ÷ 7 = ________

2,189
− ____ ← 7 × 300
89
− ____ ← 7 × 10
19
− ____ ← 7 × 2

Think•Pair•Share

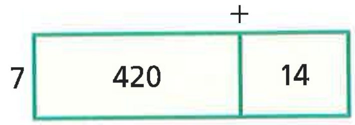
MP2 **7.** A snack company has 1,102 granola bars to give out as samples. A sample pack has 4 bars. Use the Distributive Property to find out how many sample packs the company can make. Then explain your answer.

Lesson 13 Divide Whole Numbers: Use Properties of Operations

Independent Practice

Use the area model and the Distributive Property to divide. Complete the model and equations.

1. 434 ÷ 7

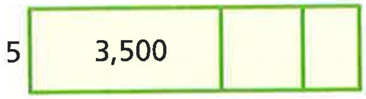


434 ÷ 7 = (____ ÷ 7) + (____ ÷ 7)

= ____ + ____

= ____

2. 3,605 ÷ 5



3,605 ÷ 5 = (________) + (________) + (________)

= ____ + ____ + ____

= ____

Use the Distributive Property to divide. Complete the equation to represent the problem. Then solve.

3. 476 ÷ 2 = (________) + (60 ÷ 2) + (________)

= ____ + ____ + 8

= ____

4. 728 ÷ 4 = (________) + (________) + (8 ÷ 4)

= ____ + 80 + ____

= ____

5. 1,932 ÷ 6 = (________) + (120 ÷ 6) + (________)

= 300 + ____ + ____

= ____

Independent Practice

Divide. Show the partial quotients. Use a strategy that works for you.

6. 830 ÷ 2 = ____

7. 675 ÷ 3 = ____

8. 2,072 ÷ 4 = ____

9. 1,968 ÷ 6 = ____

Match each of the following.

10. (800 ÷ 4) + (28 ÷ 4) ____	**a.** 855 ÷ 5
11. (5 × 100) + (5 × 30) + (5 × 2) + 1 ____	**b.** 661 ÷ 5
12. (500 ÷ 5) + (300 ÷ 5) + (55 ÷ 5) ____	**c.** 774 ÷ 4
13. (4 × 100) + (4 × 90) + (4 × 3) + 2 ____	**d.** 828 ÷ 4

Independent Practice

MP7 **14.** Look at exercises 10–13. Which of the quotients have a remainder? Explain how you know. Use the words *dividend*, *divisor*, and *addend* in your answer.

MP8 **15.** To divide 144 by 3, Pranav subtracts 3 from 144 until he has a difference of 0. Then he counts the number of times he subtracted 3. How can Pranav divide more efficiently? Use your method to find the quotient.

Solve the problems. You may use any method to divide.

MP4 **16.** Laura joins a walkathon to help raise money for a charity. The walkathon distance is 80 blocks. If you assume that 80 blocks is the same as 5 miles, how many blocks are in 1 mile?

Show your work.

Answer ______________________

MP2 **17.** Ben has 769 songs on his computer that he wants to save on CDs. He has 6 blank CDs and wants to save the same number of songs on each CD. How many songs can Ben save on each CD?

Show your work.

Answer ______________________

Independent Practice

MP8 **18.** A 7-night vacation package costs $2,583 altogether. What is the cost for each night?

Show your work.

Answer ______________________

MP2 **19.** Yuna says that 9 times as many people are at the football game as at the basketball game. The ticket office sold 7,052 tickets to the football game. If Yuna is correct, about how many people are at the basketball game?

Show your work.

Answer ______________________

MP4 **20.** Carrie divides 4,981 by 4 and finds the correct quotient, 1,245 R1. Then she represents the problem with the equation below.

$$4{,}981 \div 4 = (4{,}400 \div 4) + (400 \div 4) + (160 \div 4) + (20 \div 4) + 1$$

Is Carrie's equation correct?

Answer ______________________

Justify your answer using words, drawings, or numbers.

MP3 **21.** Sean uses place value to find 5,672 ÷ 8 with the method shown at the right. He says that he also uses the Distributive Property with this method. Do you agree with Sean?

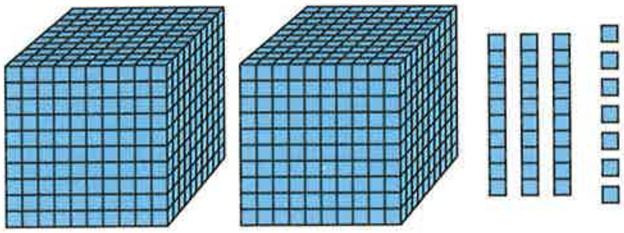
$$\begin{array}{r} 709 \\ 8\overline{)5{,}672} \\ -\,5\,6\downarrow\downarrow \\ \hline 072 \\ -\,72 \\ \hline 0 \end{array}$$

Answer ______________________

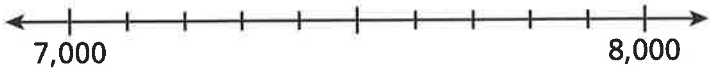
Justify your answer using words, drawings, or numbers.

UNIT 2 Common Core Review

1. What number is represented by the model below? Explain the value of each digit in the number.



2. Draw a point on the number line to show 7,352. Then round 7,352 to the nearest hundred.



Answer To the nearest hundred, 7,352 rounds to ________.

Estimate. Then add or subtract.

3. $\begin{array}{r} 4{,}389 \\ \underline{+7{,}915} \end{array}$

4. $\begin{array}{r} 112{,}075 \\ \underline{+\ \ 38{,}472} \end{array}$

5. $\begin{array}{r} 581{,}227 \\ \underline{+378{,}978} \end{array}$

6. $\begin{array}{r} 9{,}302 \\ \underline{-1{,}842} \end{array}$

7. $\begin{array}{r} 53{,}891 \\ \underline{-48{,}089} \end{array}$

8. $\begin{array}{r} 827{,}354 \\ \underline{-609{,}378} \end{array}$

Circle the correct answer.

9. Which is the number name for 805,392?

   a. eight hundred thousand five, three nine two

   b. eight hundred fifty thousand, three hundred ninety-two

   c. eight hundred five thousand, three hundred ninety-two

   d. eight hundred five thousand, thirty-nine two

10. Which of these is a different number from the others?

   a. 30,000 + 7,000 + 40 + 5

   b. 3 ten thousands + 7 thousands + 4 tens + 5 ones

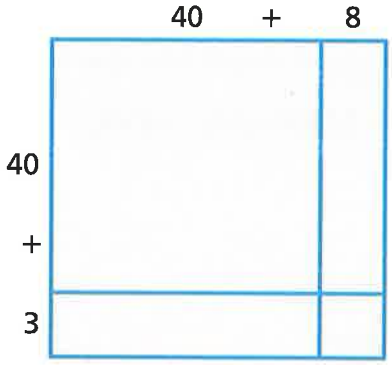
   c. 37,045

   d. thirty-seven thousand, four hundred five

UNIT 2 Common Core Review

11. Complete the area model at the right. Then multiply.

43 × 48 = ________



12. Use the Distributive Property and expanded form to multiply.

4 × 3,725 = 4 × (________ + ________ + ________ + ________)

= (________) + (________) + (________) + (________)

= ________ + ________ + ________ + ________

= ________

Use division patterns with zeros to find each quotient.

13. 600 ÷ 6 = ________ **14.** 9,000 ÷ 3 = ________ **15.** 2,400 ÷ 8 = ________

16. For a report on the world's longest rivers, Justine finds that the Lena River in Russia is 2,734 miles long and the Amur River in northeast Asia is 2,744 miles long. Which river is longer? ________________

Solve the problems.

MP4 **17.** The Obricki family is buying a computer for $765. To pay for it, they will make 9 monthly payments of the same amount. How much will one month's payment be?

Show your work.

Answer ________________________

UNIT 2 Common Core Review

MP1 **18.** The ruby-throated hummingbird beats its wings 52 times per second. How many times does the hummingbird beat its wings per minute?

Show your work.

Answer ____________________

MP3 **19.** Lennon found the sum of the following two numbers.

$$\begin{array}{r} {}^{1} \\ 57,024 \\ +\,38,983 \\ \hline 95,907 \end{array}$$

What is Lennon's mistake? What is the correct sum?

Justify your answer using words, drawings, or numbers.

Answer ____________________

MP2 **20.** Charlotte and Finn round the same number to different places. Charlotte rounds to the nearest thousand and gets 0. Finn rounds to the nearest hundred and gets 400. If they both rounded correctly, what are the greatest and least possible numbers they could have rounded?

Justify your answer using words, drawings, or numbers.

least number __________

greatest number __________

ProgressCheck

UNIT 3

Look at how the math concepts and skills you have learned and will learn connect.

It is very important for you to understand the math concepts and skills from the prior grade level so that you will be able to develop an understanding of fractions in this unit and be prepared for next year. To practice your skills, go to sadlierconnect.com.

GRADE 3 I Can...	Before Unit 3	GRADE 4 Can I ?	After Unit 3	GRADE 5 I Will...
Find equivalent fractions	☐	Explain why fractions are equivalent	☐	
Compare fractions with the same numerator or denominator	☐	Compare fractions with different numerators and denominators	☐	
Understand the meaning of the numerator of a fraction	☐	Add and subtract fractions and mixed numbers with like denominators	☐	Add and subtract fractions and mixed numbers
Understand the meaning of the denominator of a fraction	☐	Solve word problems involving addition and subtraction of fractions with like denominators	☐	Solve word problems involving addition and subtraction of fractions
Represent unit fractions on a number line	☐	Multiply a unit fraction by a whole number	☐	Multiply a fraction or whole number by a fraction
Represent fractions on a number line	☐	Multiply a fraction by a whole number	☐	Solve word problems involving multiplication of fractions and mixed numbers
	☐	Solve word problems involving multiplication of fractions by whole numbers	☐	Divide with unit fractions
Find equivalent fractions	☐	Add two fractions with denominators of 10 and 100	☐	Add and subtract decimals to hundredths
	☐	Express as decimals, fractions with denominators of 10 or 100	☐	Read and write decimals to thousandths
	☐	Compare two decimals to hundredths using $>$, $=$, or $<$	☐	Compare two decimals to thousandths

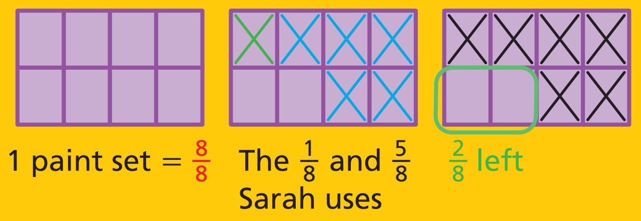
HOME ✦ CONNECT...

Models can help your child to visualize and solve problems. Visuals model are effective tools that can help your child to better understand fraction concepts. A visual model of a fraction should show the number of equal parts as indicated by the denominator of the fraction.

numerator	number of equal parts you are talking about
denominator	total number of equal parts in the whole

Support your child by using visual models such as the following when solving problems involving fractions.

Sarah has a watercolor paint set with 8 colors. She uses $\frac{1}{8}$ *of the colors first. Then she uses another* $\frac{5}{8}$ *of the colors. How much of the paint set is left?*


1 paint set = $\frac{8}{8}$
The $\frac{1}{8}$ and $\frac{5}{8}$ Sarah uses
$\frac{2}{8}$ left

Activity: Coins values are fractional parts of a dollar. A dime is $\frac{1}{10}$ of a dollar and a penny is $\frac{1}{100}$ of a dollar. While you are shopping with your child, find prices that are less than one dollar. Ask your child to tell you how many dimes and/or pennies each price represents. Then ask your child to name each price as a fraction with either 10 or 100 as the denominator.

In this unit your child will:

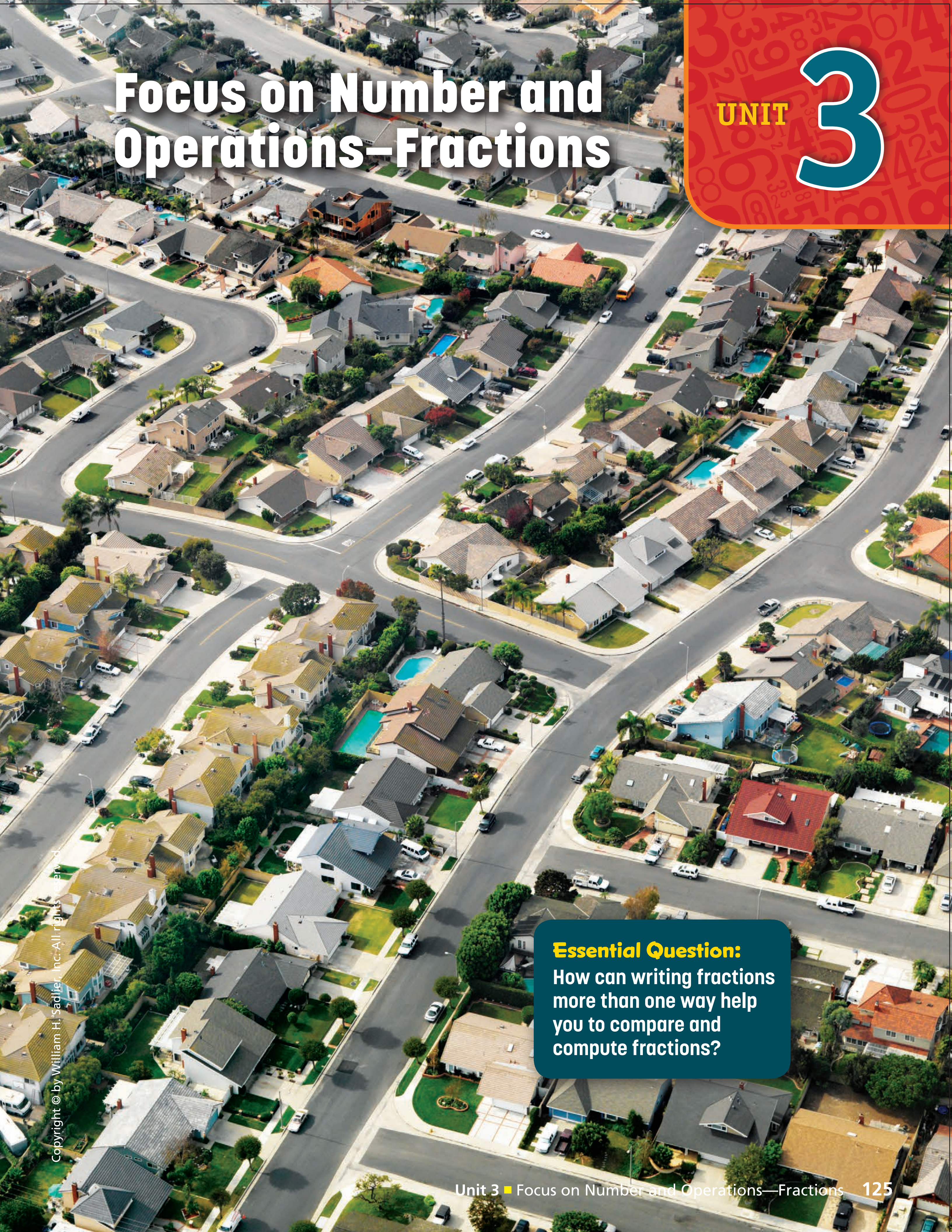
- Understand and write equivalent fractions.
- Compare fractions.
- Add and subtract fractions with like denominators.
- Add and subtract mixed numbers with like denominators.
- Multiply fractions by whole numbers.
- Write and compare decimal fractions.

Ways to Help Your Child

Manipulatives are objects that students can use to see relationships and solve problems. Virtual manipulatives exist digitally. They can also be a great tool to help your child visualize concepts. Libraries of virtual manipulatives are available online.

ONLINE

For more Home Connect activities, continue online at sadlierconnect.com



UNIT

3

Focus on Number and Operations–Fractions

Essential Question:

How can writing fractions more than one way help you to compare and compute fractions?

Lesson

14 Understand Equivalent Fractions

Essential Question:
Why can different fractions name the same amount?

Words to Know:
unit fraction
fraction
equivalent fractions

Guided Instruction

In this lesson you will learn how to model and recognize equivalent fractions.

Understand: Model equivalent fractions

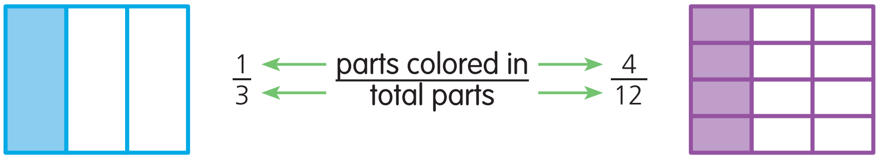
Dina's mother says that one of her plants is $\frac{1}{3}$-foot taller than last year. Dina agrees, since she measured the plant growth as $\frac{4}{12}$ foot. Are $\frac{1}{3}$ and $\frac{4}{12}$ equivalent fractions?

You can model $\frac{1}{3}$ and $\frac{4}{12}$ to decide whether they are equivalent fractions.
To compare fractions, they must have the same whole.
When a whole is partitioned, or divided, into equal parts, one of those parts is a unit fraction.
A number formed by putting together unit fractions is a fraction.
Fractions that describe the same amount are equivalent fractions.

Draw 2 same-size rectangles to represent the whole.

Divide one into 3 equal-size parts.
Color 1 part to represent $\frac{1}{3}$.

Divide the other into 12 equal-size parts.
Color 4 parts to represent $\frac{4}{12}$.



Compare the areas that you colored in. Both cover the same area of the rectangle, so the fractions they represent are equivalent.

▶ The models show that $\frac{1}{3}$ and $\frac{4}{12}$ are equivalent fractions.

Look at the area models above.

You can see that the whole with 12 equal parts has 4 times as many total parts as the whole with 3 equal parts.

$$\begin{matrix}\text{numerator} \longrightarrow \\ \text{denominator} \longrightarrow\end{matrix} \frac{1}{3} = \frac{4 \times 1}{4 \times 3} = \frac{4}{12}$$

You can see that to cover the same area as $\frac{1}{3}$ with the smaller twelfths unit fractions, you colored in 4 times as many twelfths as thirds.

When you multiply both the numerator and the denominator by the same number you get an equivalent fraction.

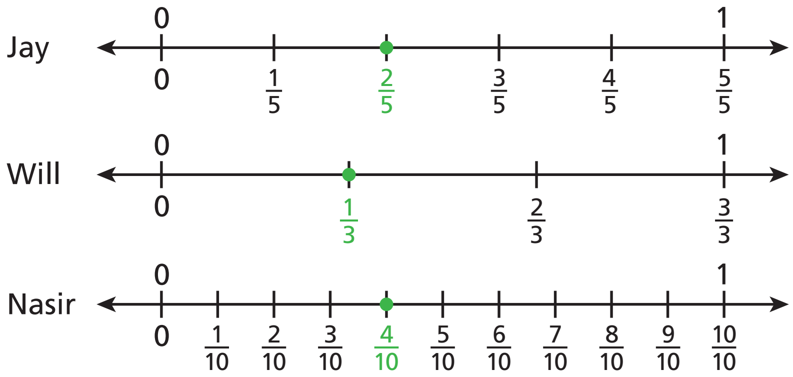
Guided Instruction

Connect: Recognize and identify equivalent fractions

Jay, Will, and Nasir have cherries for a snack. Jay has $\frac{2}{5}$ pound, Will has $\frac{1}{3}$ pound, and Nasir has $\frac{4}{10}$ pound. Which friends have the same amount of cherries?

Remember! Equivalent fractions name the same point on a number line.

To find which amounts are the same, you can use a number line to decide whether any of the fractions are equivalent.



Look at the number line models. The distance from 0 to 1 is the whole.

Use the model to identify the equivalent fractions.

$\frac{2}{5}$ and $\frac{4}{10}$ are the same distance from 0. They name the same point on the number line, so they are equivalent fractions.

$\frac{1}{3}$ is a different point on the number line. It is closer to 0 than both $\frac{2}{5}$ and $\frac{4}{10}$, so it is not equivalent to the other two fractions.

equivalent fractions

$\frac{2}{5} = \frac{4}{10}$

not equivalent fractions

$\frac{1}{3} \neq \frac{2}{5}, \frac{1}{3} \neq \frac{4}{10}$

Jay and Nasir have the same amount of cherries, because $\frac{2}{5}$ and $\frac{4}{10}$ are equivalent fractions.

Use the number line to look at the relationship between unit fractions of $\frac{1}{5}$ and unit fractions of $\frac{1}{10}$. The whole has 2 times as many $\frac{1}{10}$ unit fractions as $\frac{1}{5}$ unit fractions, so 1 unit fraction of $\frac{1}{5}$ is equal to 2 unit fractions of $\frac{1}{10}$. So, $\frac{1}{5} = \frac{2}{10}$.

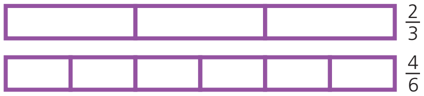
The two fractions are equivalent if you can multiply both the numerator and the denominator of one fraction by the same number to get the other fraction.

Look at $\frac{4}{5}$ and $\frac{8}{10}$. Recall that $10 = 2 \times 5$. Multiplying the numerator and denominator of $\frac{4}{5}$ by 2 you get $\frac{8}{10}$, so $\frac{4}{5}$ and $\frac{8}{10}$ are equivalent fractions.

$$\frac{4}{5} = \frac{2 \times 4}{2 \times 5} = \frac{8}{10}$$

Guided Practice

For exercises 1–4, use the model at the right.



1. Each model represents 1 whole.
Color the models to show $\frac{2}{3}$ and $\frac{4}{6}$.

2. Write = or ≠ to tell if the fractions are equivalent.

$\frac{2}{3}$ ____ $\frac{4}{6}$

3. Complete the sentences to explain the relationship.

The fractions name the __________ area.

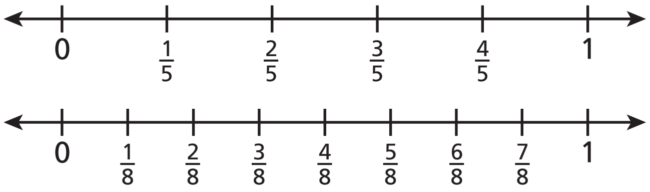
One whole has ____ times as many $\frac{1}{6}$ unit fractions as $\frac{1}{3}$ unit fractions.

Each $\frac{1}{3}$-unit fraction is equal to ____ $\frac{1}{6}$-unit fractions.

4. Write the number you can use to show that $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent.

$\frac{2}{3} = \frac{ \times 2}{ \times 3} = -$

For exercises 5–8, use the number lines at the right.



5. Place points on the number lines to show $\frac{2}{5}$ and $\frac{3}{8}$.

6. Write = or ≠ to tell whether the fractions are equivalent.

$\frac{2}{5}$ ____ $\frac{3}{8}$

7. Complete the sentences.

The fractions name __________ points on the number line.

There is no whole number of __________ that equal $\frac{2}{5}$.

There is no whole number of __________ that equal $\frac{3}{8}$.

8. Is there a whole number, n, that you can use to find an equivalent fraction?

$$\frac{2}{5} = \frac{n \times 2}{n \times 5} = \frac{3}{8}$$

Try some whole numbers.

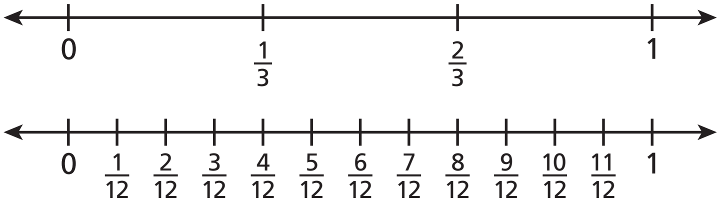
Answer ____________________

Guided Practice

Model the fractions. Write = or ≠. If the fractions are equivalent, write the number you can use to show they are equivalent.

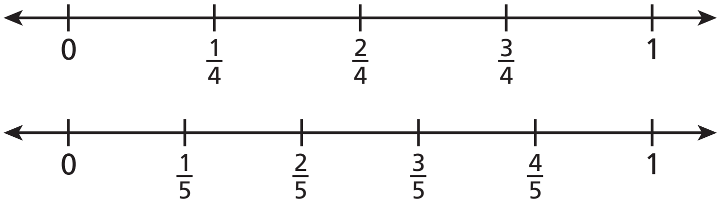
9. $\frac{2}{3}$ ____ $\frac{8}{12}$

$\frac{2}{3} = \frac{\times 2}{\times 3} = \frac{8}{12}$


0
$\frac{1}{3}$
$\frac{2}{3}$
1
0
$\frac{1}{12}$
$\frac{2}{12}$
$\frac{3}{12}$
$\frac{4}{12}$
$\frac{5}{12}$
$\frac{6}{12}$
$\frac{7}{12}$
$\frac{8}{12}$
$\frac{9}{12}$
$\frac{10}{12}$
$\frac{11}{12}$
1

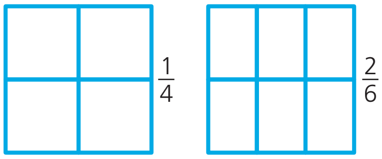
10. $\frac{3}{4}$ ____ $\frac{2}{5}$

$\frac{2}{5} = \frac{\times 2}{\times 3} = \frac{3}{4}$


0
$\frac{1}{4}$
$\frac{2}{4}$
$\frac{3}{4}$
1
0
$\frac{1}{5}$
$\frac{2}{5}$
$\frac{3}{5}$
$\frac{4}{5}$
1

11. $\frac{1}{4}$ ____ $\frac{2}{6}$

$\frac{1}{4} = \frac{\times 1}{\times 4} = \frac{2}{6}$


$\frac{1}{4}$
$\frac{2}{6}$

Write = or ≠. If the fractions are equivalent, write the number you can use to show they are equivalent. You can use models to help.

12. $\frac{1}{2}$ ____ $\frac{3}{6}$

$\frac{1}{2} = \frac{\times 1}{\times 2} = \frac{3}{6}$

13. $\frac{5}{4}$ ____ $\frac{10}{8}$

$\frac{5}{4} = \frac{\times}{\times} = \frac{10}{8}$

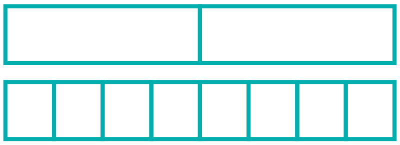
Think • Pair • Share

MP3 **14.** Pick a pair of fractions from the previous exercises that are not equivalent. Explain why the fractions are not equivalent.

Independent Practice

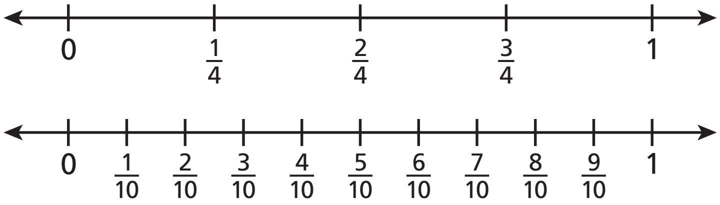
Use the models to show the fractions. Write = or ≠. If the fractions are equivalent, write the number you can use to show they are equivalent.

1. $\frac{1}{2} \underline{\qquad} \frac{4}{8}$



$\frac{1}{2} = \frac{\times 1}{\times 2} = \frac{4}{8}$

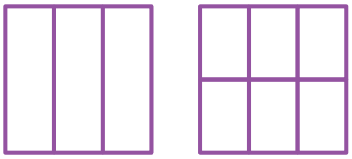
2. $\frac{2}{4} \underline{\qquad} \frac{4}{10}$



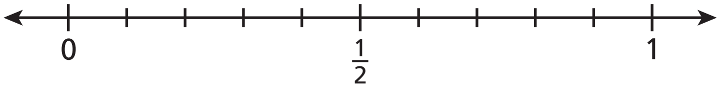
$\frac{2}{4} = \frac{\times 2}{\times 4} = \frac{4}{10}$

3. $\frac{1}{3} \underline{\qquad} \frac{2}{6}$

$\frac{1}{3} = \frac{\times 1}{\times 3} = \frac{2}{6}$

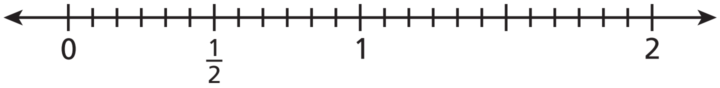


4. $\frac{4}{5} \underline{\qquad} \frac{9}{10}$



$\frac{4}{5} = \frac{\times 4}{\times 5} = \frac{9}{10}$

5. $\frac{8}{6} \underline{\qquad} \frac{16}{12}$



$\frac{8}{6} = \frac{\times 8}{\times 6} = \frac{16}{12}$

Write = or ≠. If the fractions are equivalent, write the number you can use to show they are equivalent. You can use models to help.

6. $\frac{2}{4} \underline{\qquad} \frac{2}{5}$

$\frac{2}{4} = \frac{\times 2}{\times 4} = \frac{2}{5}$

7. $\frac{2}{10} \underline{\qquad} \frac{1}{5}$

$\frac{1}{5} = \frac{\times 1}{\times 5} = \frac{2}{10}$

8. $\frac{1}{4} \underline{\qquad} \frac{3}{12}$

$\frac{1}{4} = \frac{\times}{\times} = \underline{\qquad}$

Independent Practice

Write = or ≠. If the fractions are equivalent, write the number you can use to show they are equivalent. You can use models to help.

9. $\frac{2}{6}$ ____ $\frac{4}{5}$

$\frac{2}{6} = \frac{\times 2}{\times 6} = \frac{4}{5}$

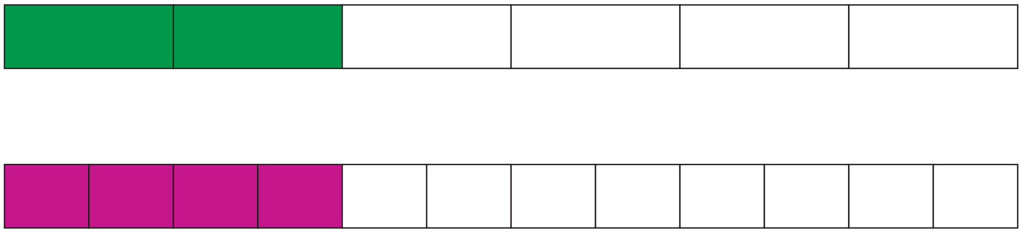
10. $\frac{6}{5}$ ____ $\frac{12}{10}$

$\frac{6}{5} = \frac{\times 6}{\times 5} = \frac{12}{10}$

11. $\frac{5}{10}$ ____ $\frac{5}{12}$

$\frac{5}{10} = \frac{\times 5}{\times 10} = \frac{5}{12}$

For exercises 12–14, use the model. Each model represents 1 whole.



12. Which pair of equivalent fractions is shown in the model?

a. $\frac{2}{4}$ and $\frac{1}{2}$ **b.** $\frac{2}{6}$ and $\frac{4}{12}$

c. $\frac{2}{4}$ and $\frac{4}{8}$ **d.** $\frac{6}{2}$ and $\frac{12}{2}$

13. Which sentence does NOT help to explain why the two fractions shown are equivalent?

a. The fractions represent the same area.

b. The fractions would share the same point on the number line.

c. There are two unit fractions of $\frac{1}{12}$ for each unit fraction of $\frac{1}{6}$.

d. One whole has more equal parts than the other.

14. Which of the fractions below are equivalent to the fractions shown in the model?

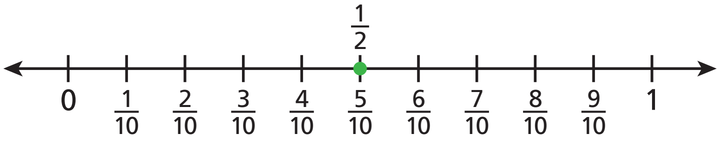
a. $\frac{1}{3}$ **b.** $\frac{2}{4}$

c. $\frac{2}{6}$ **d.** $\frac{1}{2}$

Independent Practice

MP2 **15.** Zach needs $\frac{3}{4}$ cup of sugar for a recipe. He does not have a $\frac{1}{4}$-cup measuring cup, so he fills a different measuring cup more times. What must be true about the size of the measuring cup Zach uses, compared to a $\frac{1}{4}$-cup? Explain.

MP6 **16.** The number line shows two equivalent fractions, $\frac{1}{2}$ and $\frac{5}{10}$.



What is the relationship between the unit fractions, $\frac{1}{2}$ and $\frac{1}{10}$? Describe both the number and size of the parts.

Solve the problems.

MP7 **17.** The students have $\frac{5}{6}$ hour for recess. Vinh says this is the same as $\frac{10}{12}$ hour. Is Vinh's thinking correct? Write = or ≠.

Show your work.

$\frac{5}{6}$ ____ $\frac{10}{12}$

Answer ______________________________

MP4 **18.** An egg carton holds 12 eggs. Lucy uses 5 of the eggs and says there is $\frac{2}{3}$ of a carton left. Is Lucy's thinking correct?

Show your work.

Answer ______________________________

Independent Practice

MP1 **19.** One lap around the track at Dennis's school is $\frac{1}{8}$ mile. When Dennis runs, he stops to rest only when he has completed one or more $\frac{1}{4}$-mile distances. Would Dennis stop to rest when he reaches $\frac{14}{8}$ miles?

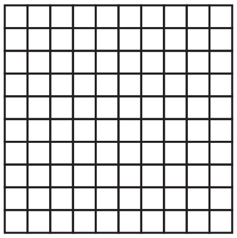
Answer ______________________________

Justify your answer using words, drawings, or numbers.

MP7 **20.** Tracy says that $\frac{30}{100}$ must be a greater amount than $\frac{3}{10}$, since 30 is greater than 3. Do you agree? You can use the model of 100 below to justify your answer.

Answer ______________________________

Justify your answer using words, drawings, or numbers.



MP3 **21.** Holly says that you cannot write equivalent fractions that have 3 and 5 as denominators. Susan says that $\frac{6}{3}$ and $\frac{10}{5}$ are equivalent fractions because they both equal 2 wholes. Whose reasoning is correct?

Show your work.

Answer ______________________________

Lesson

15 Write Equivalent Fractions

Essential Question:
How can you multiply to write equivalent fractions?

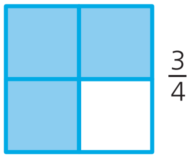
Guided Instruction

In this lesson you will learn how to write equivalent fractions.

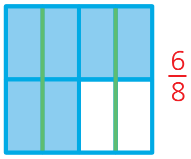
Understand: Multiply to write equivalent fractions

An apple bread recipe calls for $\frac{3}{4}$ cup of apple sauce. Cheng only has an $\frac{1}{8}$-cup measuring cup. How many times does Cheng need to fill the $\frac{1}{8}$ cup to measure $\frac{3}{4}$ cup?

To answer this question, find an equivalent fraction with a denominator of 8.



Draw an area model.
$\frac{3}{4}$ is 3 of 4 equal parts, or fourths.
To have a denominator of 8, you need to partition, or divide, the whole into 8 equal parts, or eighths.



Draw lines in the model to make 8 equal parts.
There are 2 eighths for each of the 1 fourths.
The 3 blue fourths equal 6 eighths.

Since $\frac{3}{4}$ and $\frac{6}{8}$ cover the same area, they are equivalent.

Cheng needs to fill the $\frac{1}{8}$ cup 6 times to measure $\frac{3}{4}$ cup.

Another way to write an equivalent fraction is to use multiplication.

Since $8 = 2 \times 4$, the equivalent fraction has 2 times the total number of equal parts in $\frac{3}{4}$. → $\frac{3}{4} = \frac{}{2 \times 4} = \frac{}{8}$

There are 2 eighths for each fourth.
Multiply 3, the number of fourths, by 2 to find the number of eighths in the equivalent fraction. → $\frac{3}{4} = \frac{2 \times 3}{2 \times 4} = \frac{6}{8}$

In the area model above, suppose you made 8 equal parts in a different way. Explain why the model will still show that $\frac{3}{4} = \frac{6}{8}$.

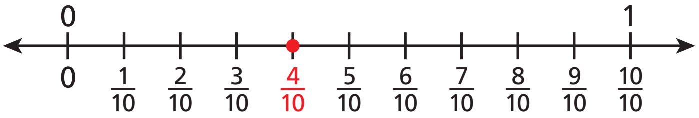
Guided Instruction

Understand: Divide to write fractions

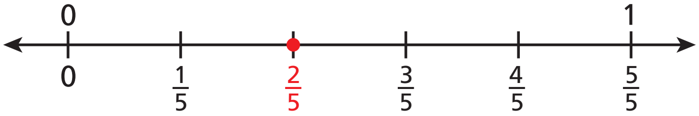
Grace wants to see 4 of the 10 movies at the theater. She tells Zena she wants to see $\frac{4}{10}$ of the movies. Zena wants to see the same movies, but she wants to use a fraction with a smaller denominator to describe the number of movies. How can Zena do this?

You can model $\frac{4}{10}$ to see whether you can find a smaller denominator.

Draw a number line with tenths. Place a point to show $\frac{4}{10}$.



Think: 10 ÷ 2 = 5. Using the same whole unit, draw another number line with fifths.



You can see that $\frac{4}{10}$ and $\frac{2}{5}$ are at the same point on the number line. It is possible to write an equivalent fraction for $\frac{4}{10}$ that has a smaller denominator.

Zena can use the number lines to write $\frac{2}{5}$ to show the number of movies she wants to see.

Another way to write an equivalent fraction is to use division.

Since 5 = 10 ÷ 2, the number of equal parts in the fraction equivalent to $\frac{4}{10}$ is 5. You start by dividing the denominator 10 by 2.

$$\frac{4}{10} = \frac{}{10 \div 2} = \frac{}{5}$$

There is 1 fifth for every 2 tenths. You find the numerator of the equivalent fraction, which represents the number of fifths, by dividing the numerator 4 by 2. By dividing the numerator and the denominator by the same number, you see that $\frac{4}{10} = \frac{2}{5}$.

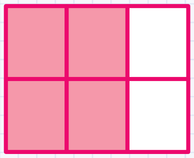
$$\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$$

Compare writing equivalent fractions by multiplying and by dividing.

Guided Instruction

Connect: What you know about equivalent fractions

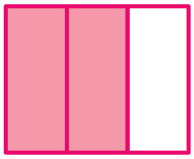
Cheng wants to give away $\frac{4}{6}$ of his apple bread. He can give bigger pieces to fewer people or smaller pieces to more people. What equivalent fractions can represent these two ways for Cheng to share the bread?



To find the answer, use models or multiply or divide both the numerator and the denominator by the same number.

If Cheng gives away $\frac{4}{6}$ of the bread which he cut into 6 pieces, he will give pieces of bread to 4 people.

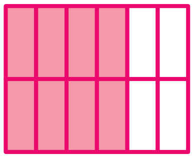
Suppose he decides to give bread to fewer people. He can divide 6 by 2 and cut the bread into only 3 pieces. To find the equivalent fraction, divide the numerator and the denominator of $\frac{4}{6}$ by 2.



$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

If Cheng gives away the same amount of bread but cuts it into 3 pieces, he will give pieces of bread to 2 people.

Suppose he decides to give bread to more people. He can multiply 6 by 2 and cut the bread into 12 pieces. To find the equivalent fraction, multiply the numerator and the denominator of $\frac{4}{6}$ by 2.



$$\frac{4}{6} = \frac{4 \times 2}{6 \times 2} = \frac{8}{12}$$

If Cheng gives away the same amount of bread but cuts it into 12 pieces, he will give pieces of bread to 8 people.

➡ Equivalent fractions representing two other ways for Cheng to share the apple bread are $\frac{2}{3}$ and $\frac{8}{12}$.

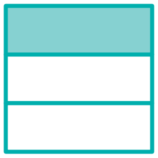
✏ Why is it important to use the same operation with the same number on both the numerator and the denominator when you want to find an equivalent fraction?

Remember!
You must multiply or divide both the numerator and the denominator by the same number.

Guided Practice

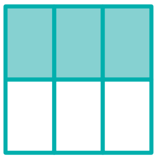
Draw on the area model to make an equivalent fraction with the given denominator. Then show what number to use to write the equivalent fraction.

1.



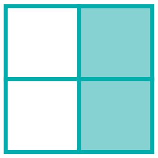
$\frac{1}{3} = \frac{\times 1}{\times 3} = \frac{\;}{6}$

2.



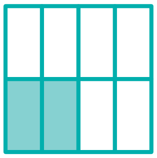
$\frac{3}{6} = \frac{\times 3}{\times 6} = \frac{\;}{12}$

3.



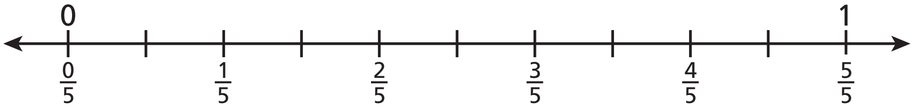
$\frac{2}{4} = \frac{2 \div}{4 \div} = \frac{\;}{2}$

4.



$\frac{2}{8} = \frac{2 \div}{8 \div} = \frac{\;}{4}$

For exercises 5–8, use the number line.



5. Write an equivalent fraction for $\frac{3}{5}$.

$\frac{3}{5} = \frac{\;}{10}$

6. Label the number line to show the fraction you wrote for exercise 5.

7. Write an equivalent fraction for $\frac{4}{10}$.

$\frac{4}{10} = \frac{\;}{5}$

8. Circle any equivalent fractions.

$\frac{1}{5}$ $\frac{1}{10}$ $\frac{2}{5}$ $\frac{2}{10}$

Think • Pair • Share

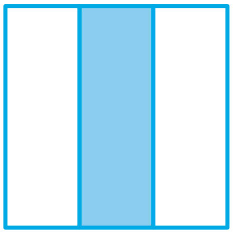
MP6 **9.** You used a number line to write an equivalent fraction for exercise 7. Explain how to use two other methods to write the equivalent fraction.

Lesson 15 Write Equivalent Fractions

Independent Practice

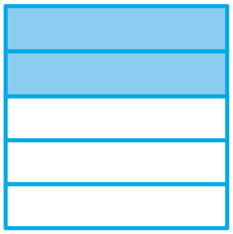
Draw on the model to make an equivalent fraction with the given denominator. For 1 and 2, show what number to use to write the equivalent fraction. For 3 and 4, complete the equation.

1.



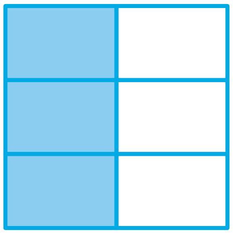
$\frac{1}{3} = \frac{\quad \times 1}{\quad \times 3} = \frac{\quad}{12}$

2.



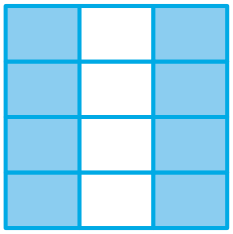
$\frac{2}{5} = \frac{\quad \times 2}{\quad \times 5} = \frac{\quad}{15}$

3.



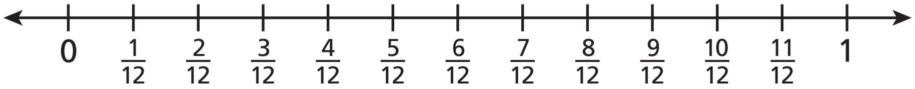
$\frac{3}{6} = \frac{\quad}{2}$

4.



$\frac{8}{12} = \frac{\quad}{3}$

Write an equivalent fraction. You can use the number line to help.



5. $\frac{6}{12} = \frac{\quad}{\quad}$

6. $\frac{8}{12} = \frac{\quad}{\quad}$

7. $\frac{5}{6} = \frac{\quad}{12}$

8. $\frac{2}{6} = \frac{4}{\quad}$

9. One point on the number line shows that three fractions are equivalent. What are the three equivalent fractions?

Independent Practice

Model the fraction. Then write an equivalent fraction.

10. $\frac{4}{8} =$ —

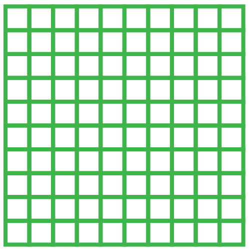
11. $\frac{6}{8} =$ —

12. $\frac{2}{12} =$ —

13. $\frac{2}{4} =$ —

14. $\frac{4}{6} =$ —

15.



$\frac{70}{100} =$ —

Show how to write an equivalent fraction. You can draw models to help.

16. $\frac{1}{5} =$ ____ = —

17. $\frac{6}{10} =$ ____ = —

18. $\frac{5}{4} =$ ____ = —

Write an equivalent fraction.

19. $\frac{10}{12} =$ —

20. $\frac{3}{12} =$ —

21. $\frac{8}{10} =$ —

22. $\frac{2}{5} =$ —

23. $\frac{9}{12} =$ —

24. $\frac{4}{10} =$ —

25. $\frac{2}{6} =$ —

26. $\frac{2}{3} =$ —

27. $\frac{5}{6} =$ —

Independent Practice

MP7 **28.** Stan says that when you multiply to get an equivalent fraction, the equivalent fraction is greater than the other fraction. Explain the error in Stan's thinking.

MP3 **29.** Dylan says that when you rename $\frac{5}{10}$ as $\frac{1}{2}$, you change the situation you are working with. Do you agree? Explain your thinking.

For exercises 30 and 31, use the table.

Soccer Goals Scored	
Player	Number of Goals
Angie	1
Kim	4
Mateo	2
Pete	3
Ricky	0
Sabrina	2

MP6 **30.** What fraction of the total number of goals did Kim score?

Show your work.

Answer ____________________

MP3 **31.** Sabrina scored $\frac{1}{6}$ of the total goals. Pete scored $\frac{1}{4}$ of the total goals. Explain why these two fractions do not have the same denominator even though they describe fractions of the same whole.

Show your work.

Answer ____________________

Independent Practice

Solve the problems.

MP4 **32.** Curtis draws a model of the fraction $\frac{5}{6}$. Then he divides each equal part into 2 smaller equal parts to help him write an equivalent fraction for $\frac{5}{6}$. What is the equivalent fraction?

Show your work.

Answer ________________________

MP2 **33.** Karl says that when you write an equivalent fraction, the size of each equal part in the equivalent fraction will always be greater than in the original fraction. Meg says this is not true, since you can multiply the numerator and denominator in a fraction to write an equivalent fraction. Whose reasoning is correct?

Answer ________________________

Justify your answer using words, drawings, or numbers.

MP6 **34.** Jack and Louise have equal-size gardens. Jack lays out his garden in 6 spaces and plants melons in 2 spaces. Louise lays out her garden in 12 spaces and plants melons in 4 spaces. Is the fraction of the garden planted with melons the same for both gardens?

Answer ________________________

Justify your answer using words, drawings, or numbers.

Lesson

16 Compare Two Fractions

Essential Question:
How can you use equivalent fractions to compare fractions?

Words to Know:
benchmark

Guided Instruction

In this lesson you will learn how to compare fractions that have different numerators and different denominators.

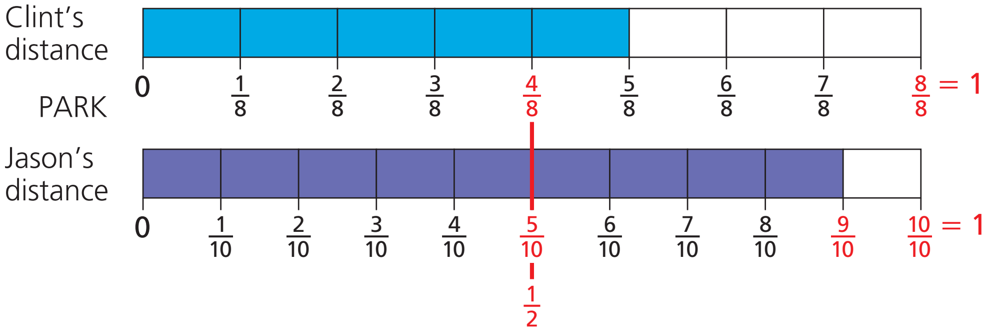
Understand: Using benchmarks to make comparisons

Clint and Jason are meeting at the park. Clint is $\frac{5}{8}$ mile away, and Jason is $\frac{9}{10}$ mile away. If the boys ride their bicycles at the same speed, who will reach the park first?

A benchmark is an amount that you know and can use to compare or estimate other amounts.

To compare $\frac{5}{8}$ and $\frac{9}{10}$, you can use $\frac{1}{2}$ and 1 as benchmarks.

Look at the models. Compare the fractions to $\frac{1}{2}$ and 1.



Notice that $\frac{4}{8} = \frac{1}{2}$, and $\frac{5}{8}$ is greater than $\frac{4}{8}$ by 1 eighth.

Clint's distance: $\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$. So $\frac{5}{8}$ is much closer to $\frac{1}{2}$ than $\frac{8}{8}$, or 1.

Notice that $\frac{10}{10} = 1$, and $\frac{9}{10}$ is less than 1 by 1 tenth.

Jason's distance: $\frac{9}{10} = \frac{10}{10} - \frac{1}{10}$. So $\frac{9}{10}$ is much closer to 1 than to $\frac{1}{2}$.

$\frac{9}{10}$ is closer to 1 on the number line than $\frac{5}{8}$ is, so $\frac{9}{10} > \frac{5}{8}$.

Jason is farther from the park, so Clint will reach the park first.

Remember!
When fractions have the same denominator, compare the numerators to see which is greater.

You can compare the fractions in this problem because both distances are fractions of a mile. They refer to the same whole amount, 1 mile.

Suppose that Clint is $\frac{7}{8}$ mile away and Jason is $\frac{9}{10}$ mile away and both travel at the same speed. Who will reach the park first?

Understand: Using equivalent fractions to make comparisons

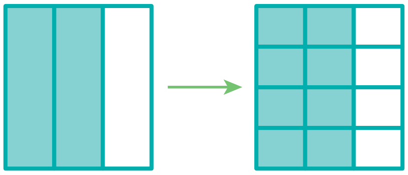
Allison has to buy clay for her art class. She is choosing between two pieces of clay. One piece is $\frac{2}{3}$ pound, and the other piece is $\frac{3}{4}$ pound. The pieces feel about the same weight. Which piece is heavier?

To decide which piece is heavier, use equivalent fractions to find the greater fraction. You can compare $\frac{2}{3}$ and $\frac{3}{4}$ because they are parts of the same whole, 1 pound.

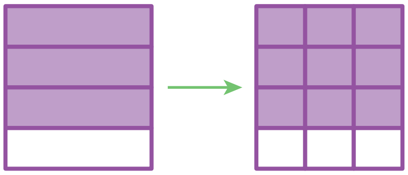
One way to compare the fractions is to write equivalent fractions with the same denominator. Use the product of the two denominators as the denominator of the equivalent fractions: $\frac{2}{3}$ and $\frac{3}{4}$; $3 \times 4 = 12$

Write equivalent fractions for $\frac{2}{3}$ and $\frac{3}{4}$ with 12 as the denominator.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$



$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$



Compare the numerators to compare the fractions.

$\frac{8}{12} < \frac{9}{12}$, so $\frac{2}{3} < \frac{3}{4}$ and $\frac{3}{4} > \frac{2}{3}$

➡ The $\frac{3}{4}$ pound piece of clay is heavier.

Another way to compare the fractions is to write equivalent fractions with the same numerator. Use the product of the numerators as the numerator of the equivalent fractions: $\frac{2}{3}$ and $\frac{3}{4}$; $2 \times 3 = 6$.

Write equivalent fractions for $\frac{2}{3}$ and $\frac{3}{4}$ with 6 as the numerator.

$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$ $\qquad$ $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

To compare the fractions, compare the denominators. Recall that unit fractions of $\frac{1}{9}$ are smaller than unit fractions of $\frac{1}{8}$.

$\frac{1}{9} < \frac{1}{8}$, so $\frac{6}{9} < \frac{6}{8}$. This means $\frac{2}{3} < \frac{3}{4}$ and $\frac{3}{4} > \frac{2}{3}$.

Guided Instruction

Connect: Benchmarks and equivalent fractions

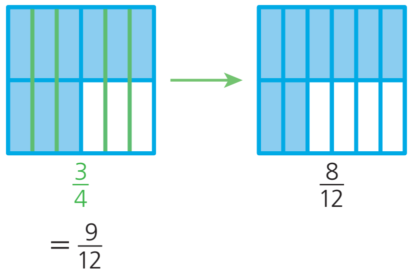
Kevin needs to buy poster board before the stores close. The drugstore closes in $\frac{3}{4}$ hour. The office supply store closes in $\frac{8}{12}$ hour. Which store is open longer?

You can use models or the same denominator to compare the fractions $\frac{3}{4}$ and $\frac{8}{12}$.

Notice that $12 = 3 \times 4$. You can use 12 as the denominator for the fractions. This means you only need to find one equivalent fraction.

Method 1 Compare with Models

Draw models of the fractions.



$\frac{9}{12}$ covers more area than $\frac{8}{12}$.

This means $\frac{9}{12} > \frac{8}{12}$, so $\frac{3}{4} > \frac{8}{12}$.

Method 2 Compare with the Same Denominator

Use 12 as the denominator to compare $\frac{3}{4}$ and $\frac{8}{12}$.
Write an equivalent fraction for $\frac{3}{4}$.

$\frac{3}{4} = \frac{3 \times 3}{3 \times 4} = \frac{9}{12}$

$\frac{9}{12} > \frac{8}{12}$, so $\frac{3}{4} > \frac{8}{12}$.

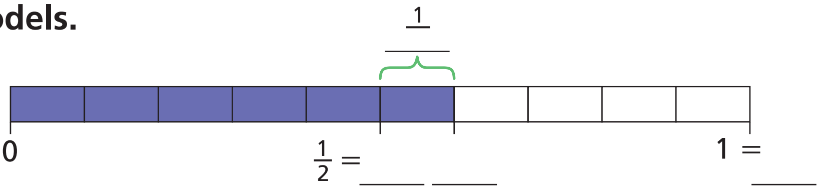
➡ The drugstore is open a little longer than the office supply store.

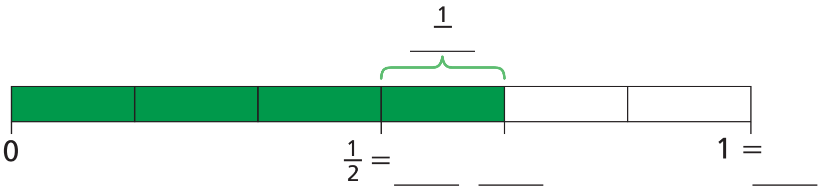
Why can you compare the two fractions to compare the amounts of time?

Guided Practice

For exercises 1–3, use the models.

1. Label the model to show:
- $\frac{6}{10}$ and $\frac{4}{6}$
- equivalent fractions for $\frac{1}{2}$
- equivalent fractions for 1
- the unit from $\frac{1}{2}$ to $\frac{6}{10}$
- the unit from $\frac{1}{2}$ to $\frac{4}{6}$





2. Circle the fraction that is farther from $\frac{1}{2}$.

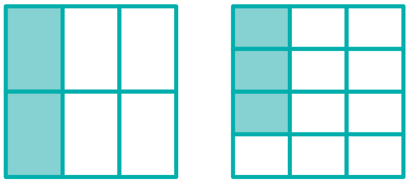
$\frac{6}{10}$ $\frac{4}{6}$

3. Write <, =, or > to compare the fractions.

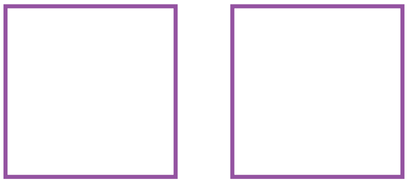
$\frac{6}{10}$ ____ $\frac{4}{6}$

Use the models to compare the fractions. Label the models to show any benchmarks you use. Then write >, =, or <.

4. $\frac{2}{6}$ ____ $\frac{3}{12}$



5. $\frac{4}{10}$ ____ $\frac{5}{8}$



Use equivalent fractions to compare.

6. Use the same numerator to compare $\frac{1}{4}$ and $\frac{2}{10}$.

$\frac{1}{4} = \frac{\times 1}{\times 4} = \frac{\_\_}{8}$ $\frac{2}{10} = \frac{\times 2}{\times 10} = \_\_$

$\_\_$ ____ $\_\_$, so $\frac{1}{4}$ ____ $\frac{2}{10}$

7. Use the same denominator to compare $\frac{9}{12}$ and $\frac{4}{5}$.

$\frac{9}{12} = \frac{\times 9}{\times 12} = \frac{45}{\_\_}$ $\frac{4}{5} = \frac{\times 4}{\times 5} = \frac{48}{\_\_}$

$\frac{45}{\_\_}$ ____ $\frac{48}{\_\_}$, so $\frac{9}{12}$ ____ $\frac{4}{5}$

Think • Pair • Share

MP8 8. Which pair of fractions from exercises 1–7 could you easily compare using a different method? Discuss your choice with a partner and explain your reasoning below.

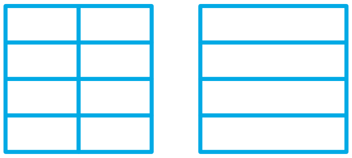
Independent Practice

Write >, =, or < to compare each fraction to $\frac{1}{2}$.

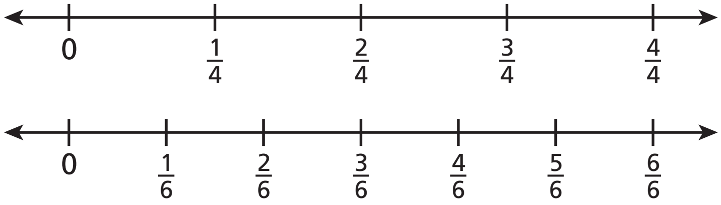
1. $\frac{8}{10}$ ____ $\frac{1}{2}$ **2.** $\frac{6}{12}$ ____ $\frac{1}{2}$ **3.** $\frac{45}{100}$ ____ $\frac{1}{2}$ **4.** $\frac{1}{1}$ ____ $\frac{1}{2}$

Color or label the models to show the fractions and any benchmarks you use. Then write >, =, or < to compare.

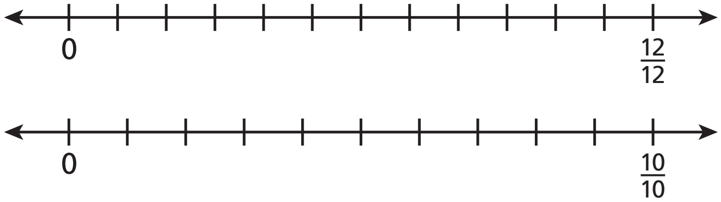
5. $\frac{6}{8}$ ____ $\frac{3}{4}$



6. $\frac{1}{4}$ ____ $\frac{2}{6}$



7. $\frac{9}{12}$ ____ $\frac{7}{10}$



Draw a model to show the fractions. Then write >, =, or < to compare.

8. $\frac{1}{6}$ ____ $\frac{2}{5}$ **9.** $\frac{2}{8}$ ____ $\frac{3}{4}$

Independent Practice

Write equivalent fractions with the same denominator or numerator to compare the fractions. Show your work.

10. Compare $\frac{5}{6}$ and $\frac{7}{12}$.

$\frac{5}{6} = \frac{\quad \times 5}{\quad \times 6} = \frac{}{}$

$\frac{}{}$ ____ $\frac{7}{12}$ $\qquad$ $\frac{5}{6}$ ____ $\frac{7}{12}$

11. Compare $\frac{3}{8}$ and $\frac{1}{3}$.

$\frac{3}{8}$ ____ $\frac{1}{3}$

12. Compare $\frac{11}{12}$ and $\frac{3}{4}$.

$\frac{11}{12}$ ____ $\frac{3}{4}$

13. Compare $\frac{5}{4}$ and $\frac{7}{5}$.

$\frac{5}{4}$ ____ $\frac{7}{5}$

Write >, =, or < to compare the fractions. You can use models to help.

14. $\frac{2}{4}$ ____ $\frac{5}{10}$

15. $\frac{3}{5}$ ____ $\frac{8}{10}$

16. $\frac{6}{8}$ ____ $\frac{4}{6}$

17. $\frac{4}{12}$ ____ $\frac{2}{5}$

18. $\frac{2}{2}$ ____ $\frac{1}{3}$

19. $\frac{9}{12}$ ____ $\frac{9}{8}$

For exercises 20 and 21, circle the correct answer.

20. Tricia has $\frac{5}{6}$ hour before she has to go to bed. How much time can she spend reading before she goes to bed?

a. $\frac{3}{4}$ hour

b. $\frac{7}{8}$ hour

c. $\frac{9}{10}$ hour

d. $\frac{11}{12}$ hour

21. Adam needs red balloons for a party. In a package of long balloons, $\frac{1}{3}$ of the balloons are red. In a package of jumbo balloons, $\frac{2}{5}$ of the balloons are red and $\frac{3}{5}$ are white. Which of these statements is true?

a. The package of jumbo balloons has more red balloons.

b. The package of long balloons has more red balloons.

c. The fractions cannot be compared without knowing the number of balloons in each package.

d. The total number of red balloons is less than the number of white jumbo balloons.

Independent Practice

MP6 **22.** Kelly uses 30 as a denominator for equivalent fractions to compare $\frac{3}{10}$ and $\frac{1}{3}$. Is this easier than using 3 as a numerator for equivalent fractions to compare $\frac{3}{10}$ and $\frac{1}{3}$? Explain.

MP1 **23.** When can writing equivalent fractions be more helpful for comparing fractions than visual models? Give one example.

Solve the problems.

MP2 **24.** In art class, Amber uses $\frac{7}{12}$ pound of modeling clay and Cole uses $\frac{5}{8}$ pound. Who uses more clay, Amber or Cole?

Show your work.

Answer ____________________

MP7 **25.** Two cereal boxes are the same size. The box of corn flakes is $\frac{11}{12}$ full. The box of wheat puffs is $\frac{9}{10}$ full. Which cereal box has more cereal?

Show your work.

Answer ____________________

Independent Practice

MP6 **26.** Jeff has $\frac{200}{100}$ dollars in pennies, and Michelle has $\frac{25}{10}$ dollars in dimes. Who has more money, Jeff or Michelle?

Show your work.

Answer ____________________

MP3 **27.** Sam says that you cannot use $\frac{1}{2}$ as a benchmark when you are comparing fractions with odd denominators, like $\frac{2}{5}$. Do you agree? Use this example to support your answer.

Answer ____________________

Justify your answer using words, drawings, or numbers.

MP1 **28.** Donna and her brother are raking leaves. Donna rakes $\frac{1}{4}$ of her yard and her brother rakes $\frac{1}{2}$ of her neighbor's yard. Donna's brother says he has raked more than Donna, because $\frac{1}{2} > \frac{1}{4}$. Is this always true?

Answer ____________________

Justify your answer using words, drawings, or numbers.

Lesson

17 Add and Subtract Fractions with Like Denominators

Essential Question:
What happens when you find a sum or difference of fractions?
4.NF.3a

Words to Know:
like denominators

Guided Instruction

In this lesson you will learn why you can add and subtract fractions of the same whole.

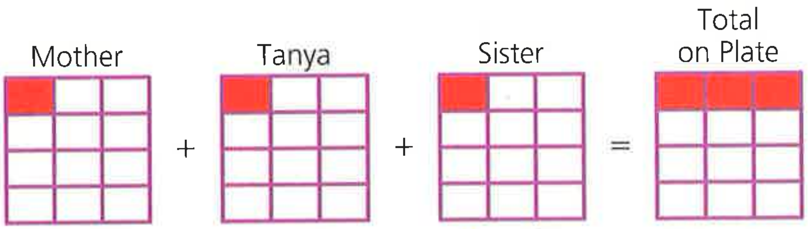
Understand: Addition of fractions with like denominators

Tanya cuts a pan of brownies into 12 equal pieces. She puts 1 piece on a plate for her mother. Then she adds 1 piece for herself, and 1 piece for her sister. What fraction of the brownies does Tanya put on the plate?

To find the fraction of the brownies Tanya put on the pan, think of each brownie as one twelfth, or $\frac{1}{12}$, of the whole pan.

Write the number of twelfths and a fraction for each number of twelfths. Then add the numerators to find the total number of twelfths.

Remember!
The numerator is the number of equal parts in a fraction.



1 twelfth + 1 twelfth + 1 twelfth = 3 twelfths

$$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12}$$

Tanya puts $\frac{3}{12}$ of the brownies on the plate.

In this problem, the addends and the sum have like denominators. This means that the denominators are alike or the same. Fractions with like denominators are made up by putting together the same unit fractions.

- Write an equation to show Tanya putting 1 more brownie on the plate.

Guided Instruction

Understand: Adding unit fractions to add fractions

Tanya and her sister drink milk with their brownies. Tanya drinks $\frac{3}{4}$ pint of milk. Her sister drinks $\frac{2}{4}$ pint of milk. How much milk do Tania and her sister drink in all?

To find the amount of milk Tania and her sister drink, add the fractions $\frac{3}{4}$ and $\frac{2}{4}$.

$$\frac{3}{4} + \frac{2}{4} = \overbrace{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}}^{\frac{3}{4}} + \overbrace{\frac{1}{4} + \frac{1}{4}}^{\frac{2}{4}}$$ Decompose the fractions into unit fractions.

$$= \frac{1 + 1 + 1 + 1 + 1}{4}$$ Add the numerators.

$$= \frac{5}{4}$$ Notice that 5 is the sum of 3 + 2.

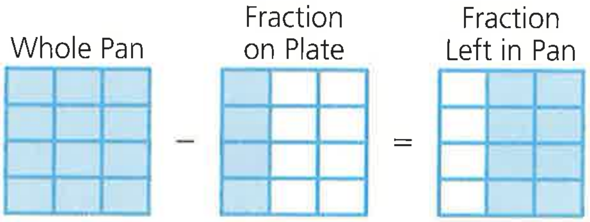
Tania and her sister drink $\frac{5}{4}$ pints of milk in all.

Understand: Subtraction of fractions with like denominators

Tanya has put a total of $\frac{4}{12}$ of the pan of brownies on a plate. What fraction of the brownies does Tanya leave in the pan?

A whole pan of brownies is $\frac{12}{12}$. Subtract $\frac{4}{12}$ from $\frac{12}{12}$ to find the number of twelfths left in the pan.

When fractions have like denominators, you can subtract the numerators to find the difference.



12 twelfths − 4 twelfths = 8 twelfths

$$\frac{12}{12} - \frac{4}{12} = \frac{8}{12}$$

Tanya leaves $\frac{8}{12}$ of the brownies in the pan.

Lesson 17 Add and Subtract Fractions with Like Denominators

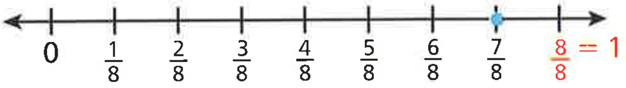
Guided Instruction

Connect: Add and subtract fractions with the same denominator

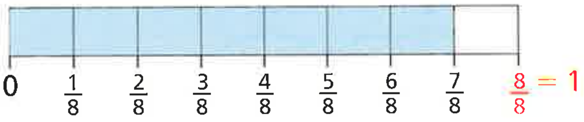
Gavin has a wood board that is $\frac{7}{8}$ yard long. If he uses only $\frac{5}{8}$ yard for his project, how much wood does Gavin have left over?

To find the length of wood left over, subtract $\frac{5}{8}$ from $\frac{7}{8}$. You can use number lines and fraction bars as models.

Identify the whole: 1 yard. Show the board length on the models.


0
$\frac{1}{8}$
$\frac{2}{8}$
$\frac{3}{8}$
$\frac{4}{8}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{7}{8}$
$\frac{8}{8} = 1$

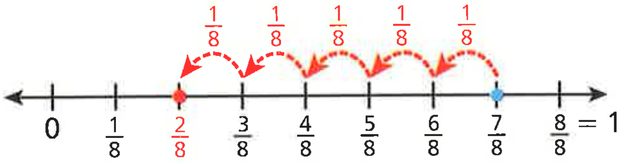
The blue point locates $\frac{7}{8}$.


0
$\frac{1}{8}$
$\frac{2}{8}$
$\frac{3}{8}$
$\frac{4}{8}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{7}{8}$
$\frac{8}{8} = 1$

The blue bar shows $\frac{7}{8}$.

Find the number of unit fractions in the amount to subtract:
$\frac{5}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

Use the number line to subtract.
Count back 5 unit fractions of $\frac{1}{8}$.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$\frac{1}{8}$
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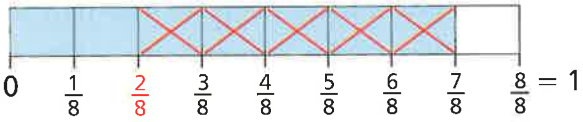
0
$\frac{1}{8}$
$\frac{2}{8}$
$\frac{3}{8}$
$\frac{4}{8}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{7}{8}$
$\frac{8}{8} = 1$

The red point locates $\frac{2}{8}$, the amount left over. The difference is $\frac{2}{8}$. $\frac{7}{8} - \frac{5}{8} = \frac{2}{8}$

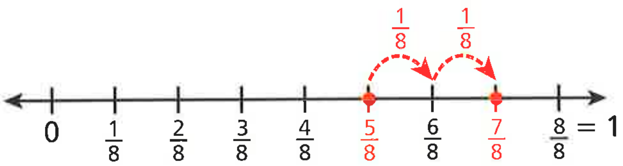
Use the fraction bar to subtract.
Cross out 5 unit fractions of $\frac{1}{8}$.


0
$\frac{1}{8}$
$\frac{2}{8}$
$\frac{3}{8}$
$\frac{4}{8}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{7}{8}$
$\frac{8}{8} = 1$

The red fraction $\frac{2}{8}$ shows the amount left over.

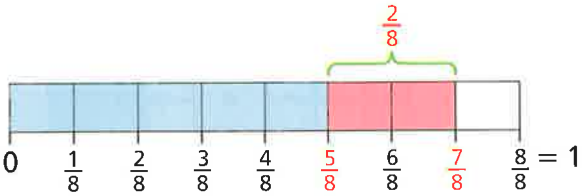
Add to check the difference, just like with whole numbers. Start at $\frac{5}{8}$.

From $\frac{5}{8}$, count up 2 unit fractions of $\frac{1}{8}$.


$\frac{1}{8}$
$\frac{1}{8}$
0
$\frac{1}{8}$
$\frac{2}{8}$
$\frac{3}{8}$
$\frac{4}{8}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{7}{8}$
$\frac{8}{8} = 1$

The sum is $\frac{7}{8}$.

From $\frac{5}{8}$, add on 2 unit fractions of $\frac{1}{8}$.


$\frac{2}{8}$
0
$\frac{1}{8}$
$\frac{2}{8}$
$\frac{3}{8}$
$\frac{4}{8}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{7}{8}$
$\frac{8}{8} = 1$

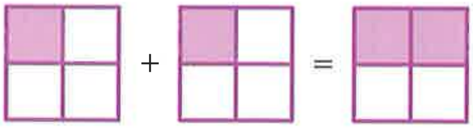
The sum is $\frac{7}{8}$.

Gavin has $\frac{2}{8}$ yard of wood left over.

Guided Practice

Complete the equation to represent the problem. Add or subtract to solve.

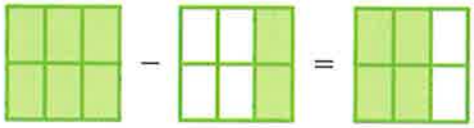
1.



$\frac{}{4} + \frac{}{4} = \frac{}{}$

1 fourth + 1 fourth = ____ fourths

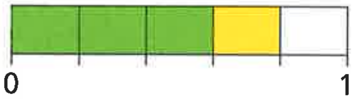
2.



$\frac{}{6} - \frac{}{6} = \frac{}{}$

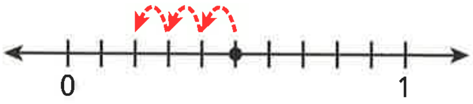
6 sixths − 2 sixths = ____ sixths

3.



$\frac{}{5} + \frac{1}{5} = \frac{}{}$

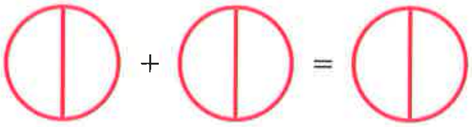
4.



$\frac{}{10} - \frac{}{10} = \frac{}{}$

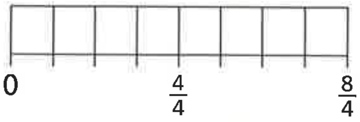
Color or draw on the model to represent the problem. Then add or subtract to solve.

5.



$\frac{1}{2} + \frac{1}{2} = \frac{}{}$

6.



$\frac{6}{4} - \frac{3}{4} = \frac{}{}$

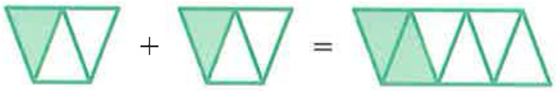
Find the sum or difference.

7. $\frac{5}{3} - \frac{1}{3} = \frac{}{}$

8. $\frac{5}{12} + \frac{9}{12} = \frac{}{}$

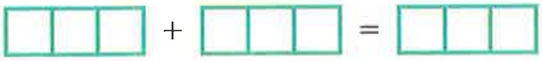
Think • Pair • Share

MP4 **9.** Lisa drew the model at the right to add $\frac{1}{3} + \frac{1}{3}$. What sum did she find? Explain Lisa's mistake.

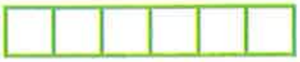


Independent Practice

Color or draw on the model to represent the problem. Then add or subtract to solve.

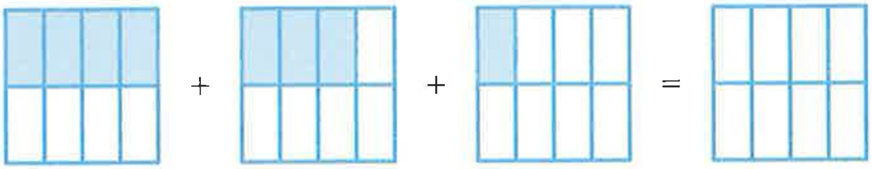
1. 

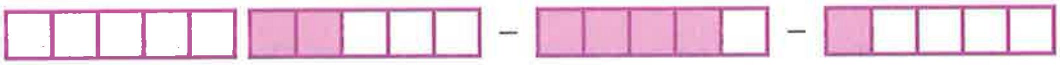
$\frac{1}{3} + \frac{1}{3} = \frac{}{}$

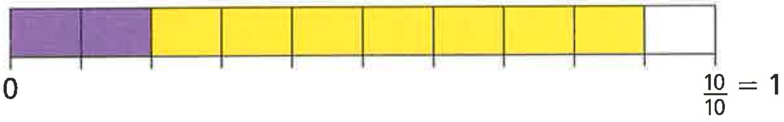
2. 

$\frac{5}{6} - \frac{2}{6} = \frac{}{}$

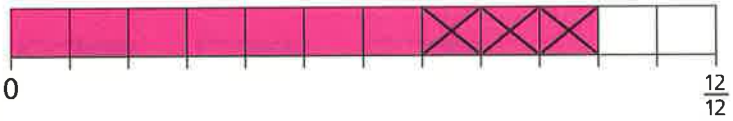
Color or draw on the model to find the sum or difference. Then write an equation to represent the problem.

3. 

4. 

5. 

6. Which difference is modeled below?



a. $\frac{7}{12} - \frac{3}{12} = \frac{4}{12}$

b. $\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$

c. $\frac{12}{12} - \frac{2}{12} = \frac{10}{12}$

d. $\frac{10}{12} - \frac{7}{12} = \frac{3}{12}$

Independent Practice

Add or subtract. Draw a model to support your answer.

7. $\frac{3}{4} - \frac{2}{4} = -$

8. $\frac{2}{2} + \frac{1}{2} = -$

9. $\frac{4}{5} + \frac{3}{5} = -$

10. $\frac{12}{10} - \frac{5}{10} = —$

Write a fraction to complete the equation. You can use models to help.

11. $\frac{5}{6} + - = \frac{6}{6}$

12. $- - \frac{3}{8} = \frac{3}{8}$

13. $— + \frac{5}{12} = \frac{13}{12}$

14. $\frac{7}{3} - - = \frac{2}{3}$

Independent Practice

MP6 **15.** Write an addition word problem that involves fractions. Then solve your problem.

MP2 **16.** Jack buys a package of gum that has 4 packs. He gives 1 pack of gum to his friend and 1 pack to his father. Explain why the equation below represents this problem.

$$\frac{4}{4} - \frac{2}{4} = \frac{2}{4}$$

Solve the problems.

MP4 **17.** There are 8 books in Audrey's favorite book series. Audrey reads 5 of the books over the winter vacation. What fraction of the series does Audrey have left to read?

Show your work.

Answer ______________________________

MP6 **18.** Kenneth walks $\frac{1}{5}$ mile to his friend's house. Then they walk $\frac{3}{5}$ mile to the ice cream store, and $\frac{2}{5}$ mile back to Kenneth's house. How far does Kenneth walk altogether?

Show your work.

Answer ______________________________

Independent Practice

MP1 **19.** Mikayla makes smoothies with $\frac{3}{4}$ pound of blueberries and $\frac{2}{4}$ pound of strawberries. Carly makes smoothies with $\frac{1}{6}$ pound of banana and $\frac{4}{6}$ pound of strawberries. Whose smoothies have more than 1 whole pound of fruit?

Show your work.

Answer ________________

MP2 **20.** Jordan answers $\frac{8}{10}$ of the questions on a quiz correctly. On her second quiz, Jordan answers $\frac{9}{10}$ of the questions correctly. If both quizzes have the same number of questions, on which quiz did Jordan have more answers wrong?

Answer ________________

Justify your answer using words, drawings, or numbers.

MP7 **21.** Oliver is choosing between two packs of baseball cards. In one pack, $\frac{8}{12}$ of the cards are for his favorite teams. In the other pack, $\frac{10}{12}$ of the cards are for his favorite teams. Oliver says he will get more cards that he likes with the second pack. Is Oliver correct?

Answer ________________

Justify your answer using words, drawings, or numbers.

Lesson

18 Decompose a Fraction as a Sum of Fractions

Essential Question:
How can you decompose a fraction or mixed number into a sum of fractions with the same denominator?

4.NF.3b

Words to Know:
decompose
mixed number

Guided Instruction

In this lesson you will learn how to decompose fractions and mixed numbers and write them as sums of other fractions.

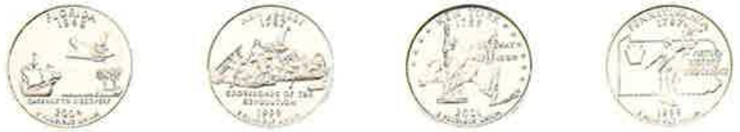
Understand: Ways to break apart a whole

Sydney collects quarters. She arranges them in rows of 4 quarters. She arranges the quarters in each row in a different way. Then she writes an equation to describe the arrangement. What are some equations she writes?

To find the equations she writes, recall that 1 quarter is $\frac{1}{4}$ of 1 dollar. This means that 4 quarters are $\frac{4}{4}$ of 1 dollar.

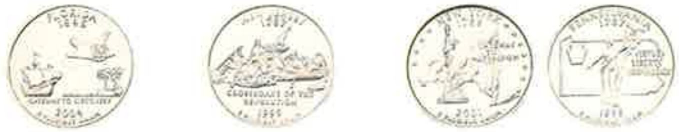
Remember!
$\frac{4}{4} = 1$, because 4 unit fractions of $\frac{1}{4}$ equal 1 whole.

She makes a row with 4 groups of 1 quarter each.



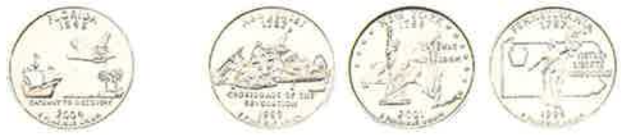
She writes $\frac{4}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

She makes a row with 2 groups of 1 quarter and 1 group of 2 quarters.



She writes $\frac{4}{4} = \frac{1}{4} + \frac{1}{4} + \frac{2}{4}$.

She makes a row with 1 group of 1 quarter and 1 group of 3 quarters.



She writes $\frac{4}{4} = \frac{1}{4} + \frac{3}{4}$.

➡ Some equations Sydney writes are $\frac{4}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, $\frac{4}{4} = \frac{1}{4} + \frac{1}{4} + \frac{2}{4}$, and $\frac{4}{4} = \frac{1}{4} + \frac{3}{4}$.

Compare the equations that Sydney wrote.

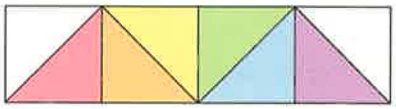
Guided Instruction

Understand: Ways to decompose a fraction

For Fred's math and art project, he draws identical rectangles with 8 equal sections. He uses 6 sections in each rectangle to represent $\frac{6}{8}$ and paints them to show ways to decompose $\frac{6}{8}$. Then he writes equations to describe his work. What kinds of sums do Fred's equations describe?

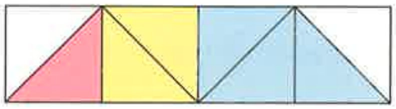
You can decompose, or break apart, a fraction by writing it as the sum of other fractions with the same denominator.

One way to decompose $\frac{6}{8}$ is to write it as the sum of unit fractions.



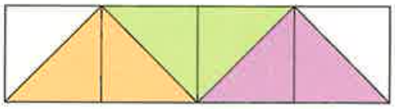
$$\frac{6}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

Another way to decompose $\frac{6}{8}$ is to write it as the sum of unit fractions and other fractions.



$$\frac{6}{8} = \frac{1}{8} + \frac{2}{8} + \frac{3}{8}$$

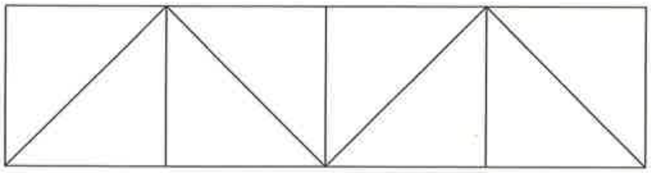
A third way to decompose $\frac{6}{8}$ is to write it as the sum of fractions that are not unit fractions.



$$\frac{6}{8} = \frac{2}{8} + \frac{2}{8} + \frac{2}{8}$$

Fred's equations describe sums of unit fractions, of unit fractions and other fractions, and of fractions that are not unit fractions.

Color the rectangle to show another way to decompose $\frac{6}{8}$. Then write an equation to describe your work.



Guided Instruction

Connect: Fractions and mixed numbers

Jason has $\frac{9}{4}$ quarts of milk. Is that more or less than 2 quarts of milk?

To decide, write $\frac{9}{4}$ as a mixed number. Then compare the mixed number to 2. A mixed number shows the sum of a whole number and a fraction but does not have a + sign.

$\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$	Write $\frac{9}{4}$ as a sum of fractions.
$= 1 + 1 + \frac{1}{4}$	Write $\frac{4}{4}$ as 1.
$= 2 + \frac{1}{4}$	Add the whole numbers.
$= 2\frac{1}{4}$	Write the sum as a mixed number.

Remember!
$\frac{4}{4} = 1$

Compare. $2\frac{1}{4} > 2$, because $2 + \frac{1}{4}$ is more than 2.

Jason has more than 2 quarts of milk.

Try a different way to compare $\frac{9}{4}$ and 2. Write 2 as a fraction and compare it with $\frac{9}{4}$. Why is the answer the same?

Ms. Gordon has $4\frac{1}{2}$ dozen eggs to make scrambled eggs for breakfast at camp. She uses $\frac{1}{2}$ dozen for each batch. How many batches can she make?

To find out, write $4\frac{1}{2}$ as a fraction with a denominator of 2. The numerator will be the number of half-dozens of eggs.

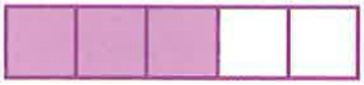
$4\frac{1}{2} = 4 + \frac{1}{2}$	Write $4\frac{1}{2}$ as a sum of 4 plus $\frac{1}{2}$.
$= 1 + 1 + 1 + 1 + \frac{1}{2}$	Write 4 as a sum of 1s.
$= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2}$	Write 1 as a fraction with the denominator 2.
$= \frac{2 + 2 + 2 + 2 + 1}{2}$	Add the numerators.

$= \frac{9}{2}$; The numerator is 9. There are 9 half-dozens of eggs.

Ms. Gordon can make 9 batches of scrambled eggs.

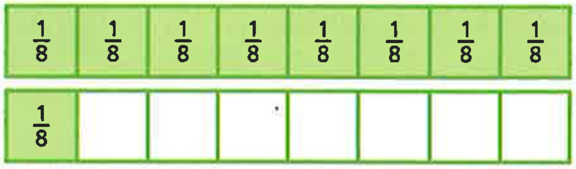
Guided Practice

Complete each equation to write the fraction as a sum of other fractions. Use the model to help you.



1. $\frac{3}{5} = \frac{}{5} + \frac{}{5} + \frac{}{5}$

$\frac{3}{5} = \frac{}{5} + \frac{}{5}$



2. $\frac{9}{8} = \frac{}{8} + \frac{}{8} + \frac{}{8} + \frac{2}{8} + \frac{1}{8}$

$\frac{9}{8} = \frac{}{8} + \frac{}{8} + \frac{}{8} + \frac{}{8}$

$\frac{9}{8} = \frac{}{8} + \frac{}{8} + \frac{}{8}$

Write each mixed number as a fraction. Show your work.

3. $2\frac{2}{3} = \frac{}{3}$ **4.** $1\frac{5}{8} = \frac{}{8}$ **5.** $4\frac{3}{5} = \frac{}{5}$

Write each fraction as a mixed number.

6. $\frac{9}{4} =$ ______ **7.** $\frac{7}{2} =$ ______ **8.** $\frac{10}{3} =$ ______

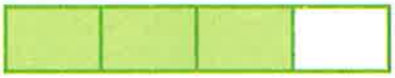
Think • Pair • Share

MP6 **9.** Michael writes $\frac{10}{12}$ as $\frac{10}{10} + \frac{2}{12}$. Draw a model to show Michael's sum. Is his sum correct? If not, what mistakes did he make?

Independent Practice

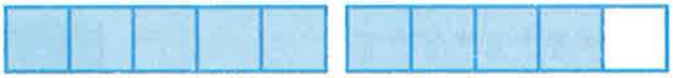
Complete each equation to write the number as a sum of other fractions in different ways. Use the model to help you.

1. $\frac{3}{4} = \frac{\ }{4} + \frac{\ }{4} + \frac{\ }{4}$



$\frac{3}{4} =$ ____ + ____

2. $1\frac{4}{5} =$ ____ $+ \frac{\ }{5}$



$1\frac{4}{5} = \frac{\ }{5} + \frac{\ }{5}$

$1\frac{4}{5} =$ ______________________

Write the fraction as a sum of other fractions in four different ways. Draw a model if it helps.

3. $\frac{9}{12}$

$\frac{9}{12} =$ ______________________

$\frac{9}{12} =$ ______________________

$\frac{9}{12} =$ ______________________

$\frac{9}{12} =$ ______________________

4. $\frac{10}{10}$

$\frac{10}{10} =$ ______________________

$\frac{10}{10} =$ ______________________

$\frac{10}{10} =$ ______________________

$\frac{10}{10} =$ ______________________

5. $\frac{12}{5}$

$\frac{12}{5} =$ ______________________

$\frac{12}{5} =$ ______________________

$\frac{12}{5} =$ ______________________

$\frac{12}{5} =$ ______________________

Independent Practice

Circle the correct answer.

6. Which sum is not equivalent to $\frac{5}{6}$?

a. $\frac{1}{6} + \frac{2}{6} + \frac{2}{6}$

b. $\frac{3}{6} + \frac{2}{6}$

c. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

d. $\frac{4}{6} + \frac{1}{6}$

7. Which sum is equivalent to $\frac{10}{3}$?

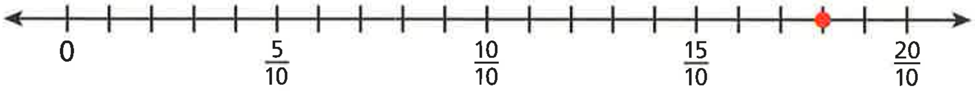
a. $3 + \frac{1}{3}$

b. $\frac{3}{3} + \frac{6}{3}$

c. $2 + \frac{3}{3}$

d. $10 + \frac{1}{3}$

8. Look at the red point on the number line. It represents a fraction. Jessica wants to write the fraction as a sum of three addends.



So far, she has two addends, $\frac{4}{10}$ and $\frac{6}{10}$. Which of the following can Jessica use as the third addend?

a. $\frac{10}{10}$

b. $\frac{18}{10}$

c. $1\frac{8}{10}$

d. $\frac{8}{10}$

Write each number as a sum in three different ways. You can use a model to help. Then answer the question below.

9. $2\frac{2}{12}$

= ____________

= ____________

= ____________

10. $\frac{22}{12}$

= ____________

= ____________

= ____________

11. Compare $2\frac{2}{12}$ and $\frac{22}{12}$. Write <, =, or >.

$2\frac{2}{12}$ ____ $\frac{22}{12}$

Independent Practice

MP3 **12.** To write $\frac{35}{10}$ as a sum, Matt says you can find addends by subtracting fractions from $\frac{35}{10}$ until there is nothing left. Does Matt's method work? Explain.

MP2 **13.** Brianna is making a patchwork quilt with different size patches. She has a strip of fabric that is $1\frac{4}{8}$ yards long. How can writing $1\frac{4}{8}$ as a sum help Brianna to cut up the strip of fabric?

Solve the problems.

MP4 **14.** For each scoop of ice cream that you buy, the ice-cream shop crosses out 1 of the 8 boxes on a card. After 4 visits, $\frac{7}{8}$ of Juan's card is crossed out. Write an addition equation to show how Juan's card could have been crossed out in 4 visits.

Show your work.

Answer ______________________

MP7 **15.** Bella serves pizza at her birthday party. Each pizza is cut into 6 equal slices. There are $\frac{13}{6}$ pizzas left after the party is over. The slices are in 3 different pizza boxes, and none of the boxes are full. What fraction of a whole pizza can be in each box?

Show your work.

Answer ______________________

Independent Practice

MP2 **16.** Travis's family uses an automatic cat feeder when they are not home. The feeder has 8 sections that can each hold 1 cup of food. Travis has $\frac{26}{4}$ cups of cat food. Does he have enough to fill each section in the feeder with a full cup?

Show your work.

Answer ______________________

MP1 **17.** A piece of wood is $4\frac{2}{6}$ yards long. If $\frac{3}{6}$ yard is needed to make 1 shelf, how many shelves can you make from the piece of wood?

Answer ______________________

Justify your answer using words, drawings, or numbers.

MP8 **18.** To find the whole amounts in $\frac{27}{5}$, Molly divides the numerator by the denominator. Then she writes the remainder as the fraction part of a mixed number. Does Molly's method work? Draw a model and write $\frac{27}{5}$ as a sum to help support your answer.

Answer ______________________

Justify your answer using words, drawings, or numbers.

Lesson

19 Add and Subtract Mixed Numbers with Like Denominators

Essential Question:
What methods can you use for adding and subtracting mixed numbers?
4.NF.3c

Guided Instruction

In this lesson you will learn different methods for adding and subtracting mixed numbers with like denominators.

Understand: Adding mixed numbers

Max is training his puppy to walk with a leash. He spends $1\frac{3}{4}$ hours at dog school. He also practices with his dog at the park for $1\frac{2}{4}$ hours. How much time does Max spend training his puppy altogether?

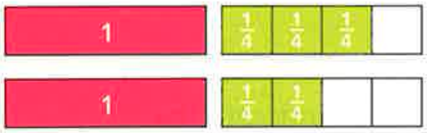
You can use models of equivalent fractions or properties of addition to find the sum of $1\frac{3}{4} + 1\frac{2}{4}$.

One way is to use models of equivalent fractions.

Model each mixed number.

$1\frac{3}{4}$

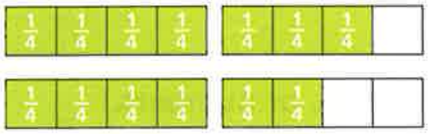
$1\frac{2}{4}$



Rename each mixed number as an equivalent fraction.

$1\frac{3}{4} = \frac{7}{4}$

$1\frac{2}{4} = \frac{6}{4}$



Add. Write the sum as a mixed number.

$$\frac{7}{4} + \frac{6}{4} = \frac{13}{4} = \frac{12}{4} + \frac{1}{4}$$
$$= 3 + \frac{1}{4} = 3\frac{1}{4}$$

Another way is to use properties of addition. Using the properties of addition, you can add mixed numbers by first breaking them apart and then adding the whole numbers and the fractions separately.

$1\frac{3}{4} + 1\frac{2}{4}$	Write the sum you want to find.
$(1 + \frac{3}{4}) + (1 + \frac{2}{4})$	Write the mixed numbers as sums.
$1 + (\frac{3}{4} + 1) + \frac{2}{4}$	Use the Associative Property.
$1 + (1 + \frac{3}{4}) + \frac{2}{4}$	Use the Commutative Property.
$(1 + 1) + (\frac{3}{4} + \frac{2}{4})$	Use the Associative Property.
$2 + \frac{5}{4}$	Add the whole numbers. Add the fractions.
$2 + 1 + \frac{1}{4}$	Rename $\frac{5}{4}$.
$3 + \frac{1}{4}$	Add the whole numbers.
$3\frac{1}{4}$	Write the mixed number.

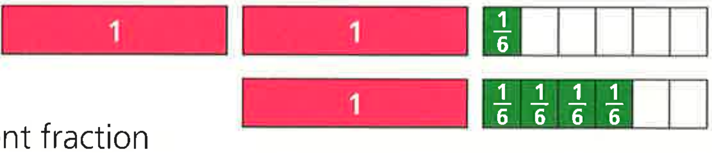
➡ Max spends $3\frac{1}{4}$ hours training his puppy.

Understand: Subtracting mixed numbers

Sofia is hanging a poster in her bedroom on the wall space between her windows. That space is $2\frac{1}{6}$ feet wide. Her poster is $1\frac{4}{6}$ feet wide. How much space is left after Sofia hangs her poster?

One way to subtract mixed numbers is to use equivalent fractions.

The models show that you need to regroup sixths to subtract $1\frac{4}{6}$ from $2\frac{1}{6}$.



Rename each mixed number as an equivalent fraction greater than 1, with 6 as the denominator.

$$2\frac{1}{6} = 1 + 1 + \frac{1}{6} = \frac{6}{6} + \frac{6}{6} + \frac{1}{6} = \frac{13}{6}$$

$$1\frac{4}{6} = 1 + \frac{4}{6} = \frac{6}{6} + \frac{4}{6} = \frac{10}{6}$$

Now subtract the fractions to find the difference.

$$2\frac{1}{6} - 1\frac{4}{6} = \frac{13}{6} - \frac{10}{6} = \frac{13 - 10}{6} = \frac{3}{6}$$

Remember!
To add or subtract fractions with like denominators, add or subtract the numerators.

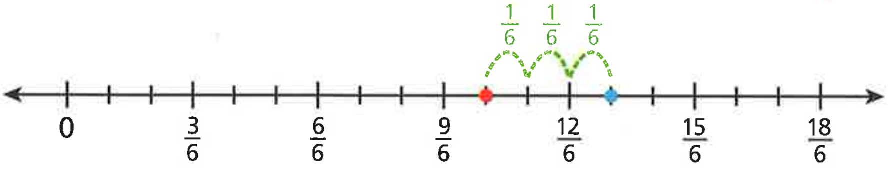
Sofia has $\frac{3}{6}$ foot of space left after she hangs the poster.

Another way to subtract is to use the relationship between addition and subtraction.

Change the subtraction problem to an addition problem with an unknown addend.

$$2\frac{1}{6} - 1\frac{4}{6} = ? \qquad 1\frac{4}{6} + ? = 2\frac{1}{6}$$

Use a number line to find the unknown addend. Locate $1\frac{4}{6}$, or $\frac{10}{6}$, and $2\frac{1}{6}$, or $\frac{13}{6}$.



The distance between them is the unknown addend: $\frac{3}{6}$.

Guided Instruction

Connect: Using methods for adding and subtracting mixed numbers

Mitch, Ruby, and Tony are in a relay race. Mitch finishes the first part of the race in $3\frac{5}{10}$ minutes. Ruby does the second part in $2\frac{7}{10}$ minutes. If the team's total race time is 9 minutes, how long does Tony take to finish the last part?

To solve, add Mitch and Ruby's times, then subtract the sum from the team's total finishing time.

Step 1

Add $3\frac{5}{10}$ and $2\frac{7}{10}$ to find Mitch and Ruby's total time.

First, rename each mixed number as an equivalent fraction.

Since $1 = \frac{10}{10}$, you can find the total number of tenths in the whole number parts of the mixed numbers.

$$3\frac{5}{10} = \frac{10 \times 3}{10 \times 1} + \frac{5}{10} = \frac{30}{10} + \frac{5}{10} = \frac{35}{10}$$

$$2\frac{7}{10} = \frac{10 \times 2}{10 \times 1} + \frac{7}{10} = \frac{20}{10} + \frac{7}{10} = \frac{27}{10}$$

Remember! You can write any whole number as a fraction with a denominator of 1.

Add the fractions. $\frac{35}{10} + \frac{27}{10} = \frac{35 + 27}{10} = \frac{62}{10}$

Step 2

Subtract $\frac{62}{10}$ from 9, the team's total time, to find Tony's time.

First, rename 9 as a fraction. $9 = \frac{10 \times 9}{10 \times 1} = \frac{90}{10}$

Subtract $\frac{62}{10}$ from $\frac{90}{10}$: $\frac{90}{10} - \frac{62}{10} = \frac{90 - 62}{10} = \frac{28}{10}$

Step 3

Rename the difference $\frac{28}{10}$ as a mixed number.

$$\frac{28}{10} = \frac{10}{10} + \frac{10}{10} + \frac{8}{10} = 1 + 1 + \frac{8}{10} = 2\frac{8}{10}$$

Remember! Make sure the fraction in the mixed number is less than 1.

▶ Tony takes $2\frac{8}{10}$ minutes to finish the last part of the race.

Guided Practice

Rename each mixed number as an equivalent fraction greater than 1. Then add or subtract.

1. Add $2\frac{1}{5}$ and $1\frac{3}{5}$.

a. Rename $2\frac{1}{5}$ as a fraction. $2\frac{1}{5} = \overbrace{\frac{}{5} + \frac{}{5}}^{\text{2 wholes}} + \frac{1}{5} = \frac{}{5}$

b. Rename $1\frac{3}{5}$ as a fraction. $1\frac{3}{5} = \frac{}{5} + \frac{3}{5} = \frac{}{5}$

c. Add the renamed fractions. $\frac{}{5} + \frac{}{5} = \frac{}{5}$

d. Write the sum as a mixed number. $\frac{19}{5} = \frac{}{5} + \frac{}{5} + \frac{}{5} + \frac{4}{5} = 3\frac{}{5}$

e. Complete the addition equation. $2\frac{1}{5} + 1\frac{3}{5} = \underline{\hspace{2em}}\frac{}{}$

2. Subtract $1\frac{4}{8}$ from $3\frac{1}{8}$.

a. To subtract $1\frac{4}{8}$ do you need to regroup $3\frac{1}{8}$? $\underline{\hspace{3em}}$

Why? Because $\frac{1}{8}$ $\underline{\hspace{2em}}$ $\frac{4}{8}$

b. Rename $1\frac{4}{8}$ as a fraction. $1\frac{4}{8} = \frac{}{8} + \frac{}{8} = \frac{}{8}$

c. Rename $3\frac{1}{8}$ as a fraction. $3\frac{1}{8} = \frac{8\times}{8\times} + \frac{}{8} = \frac{}{8} + \frac{1}{8} = \frac{}{8}$

d. Subtract the renamed fractions. $\underline{\hspace{2em}} - \underline{\hspace{2em}} = \underline{\hspace{2em}}$

e. Write the difference as a mixed number. $\frac{}{8} = \frac{}{8} + \frac{}{8} = \underline{\hspace{2em}}\frac{}{}$

f. Complete the subtraction equation. $3\frac{1}{8} - 1\frac{4}{8} = \underline{\hspace{2em}}\frac{}{}$

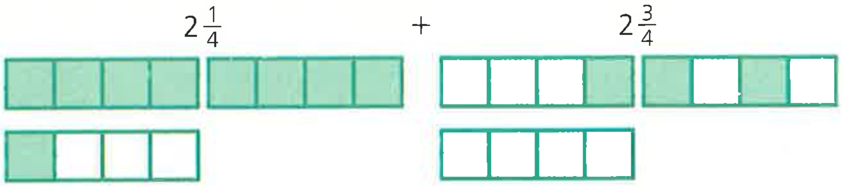
3. Find the sum $5\frac{2}{12} + 2\frac{11}{12}$. Use the properties of addition to break apart the numbers to add. Show your work.

Think • Pair • Share

MP7 **4.** Compare the methods you used to add in exercises 1 and 3. Which method is easier for you? Why?

Independent Practice

For exercises 1 and 2, use the model below.



1. Break apart $2\frac{1}{4}$ and $2\frac{3}{4}$. Then use properties to add.

$2\frac{1}{4} + 2\frac{3}{4} = \underline{\quad} + \frac{}{4} + \underline{\quad} + \frac{}{}$

$= (\underline{\quad} + \underline{\quad}) + \frac{}{4} + \frac{}{4}$

$= \underline{\quad} + \frac{}{4}$

$= \underline{\quad} + \underline{\quad}$

$= \underline{\quad}$

$2\frac{1}{4} + 2\frac{3}{4} = \underline{\quad}$

2. Rename $2\frac{1}{4}$ and $2\frac{3}{4}$ as equivalent fractions greater than 1.

$2\frac{1}{4} = \frac{4\times}{4\times} + \frac{}{4} = \frac{}{4} + \frac{}{4} = \frac{}{4}$

$2\frac{3}{4} = \frac{4\times}{4\times} + \frac{}{4} + \frac{}{4} + \frac{}{4}$

$= \frac{}{4} + \frac{}{4} + \frac{}{4} + \frac{}{4} = \frac{}{4}$

Add the fractions.

$\frac{}{4} + \frac{}{4} = \underline{\qquad\qquad}$

$2\frac{1}{4} + 2\frac{3}{4} = \underline{\quad}$

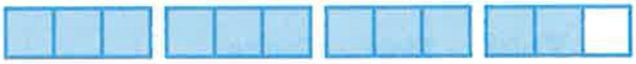
Add with the method of your choice. Show your work.

3. $6\frac{1}{2} + 3\frac{1}{2} = \underline{\quad}$

4. $4\frac{2}{6} + 2\frac{3}{6} = \underline{\quad}$

Find the difference. Use the model if it helps.

5. $3\frac{2}{3} - 2\frac{1}{3}$



Do you need to regroup? ________

Subtract the whole numbers.

____

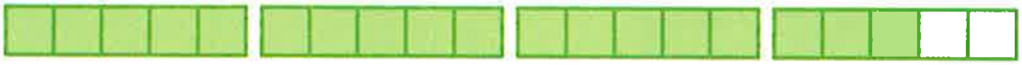
Subtract the fractions.

$\frac{2}{3} - \frac{1}{3} = \frac{}{}$

$3\frac{2}{3} - 2\frac{1}{3} = \underline{\quad}$

Independent Practice

6. $3\frac{3}{5} - 2\frac{4}{5}$ Do you need to regroup? __________



Rename each number as an equivalent fraction greater than 1.

$3\frac{3}{5} = \frac{5 \times}{5 \times} + \frac{}{5} = \frac{}{5} + \frac{}{5} = \frac{}{5}$ $2\frac{4}{5} = \frac{5 \times}{5 \times} + \frac{}{5} = \frac{}{5} + \frac{}{5} = \frac{}{5}$

Subtract the fractions: $\frac{}{5} - \frac{}{5} =$ ____.

Use the model to show your work.

$3\frac{3}{5} - 2\frac{4}{5} =$ ____

Subtract with the method of your choice. Show your work.

7. $8\frac{7}{10} - 5\frac{2}{10} =$ __________

8. $3\frac{2}{5} - 1\frac{4}{5} =$ __________

Circle the correct answer or answers.

9. For which of the following subtraction problems do you need to regroup?

a. $5 - 2\frac{1}{2}$

b. $4\frac{6}{8} - 3\frac{5}{8}$

c. $10\frac{1}{5} - 8\frac{3}{5}$

d. $2\frac{50}{100} - 1\frac{9}{100}$

10. Penny is mailing two packages. To find their total weight, she is adding $8\frac{5}{12}$ pounds and $3\frac{7}{12}$ pounds. Which of the following is the best way to group the addends to add mentally?

a. $8 + \frac{5}{12} + 3 + \frac{7}{12}$

b. $3 + \frac{7}{12} + 8 + \frac{5}{12}$

c. $8 + 3 + (\frac{5}{12} + \frac{7}{12})$

d. $8 + 3 + (\frac{5}{12} + \frac{5}{12}) + \frac{2}{12}$

Independent Practice

MP2 **11.** How is adding and subtracting mixed numbers different from adding and subtracting fractions less than 1?

MP7 **12.** When you break apart a mixed number to add, does it matter if you add the whole numbers first or the fractions first? Explain.

Solve the problems.

MP6 **13.** Greg is making homemade pizzas. For the crust, he uses $2\frac{2}{4}$ cups of whole wheat flour and $4\frac{2}{4}$ cups of regular flour. How much flour does Greg use altogether?

Show your work.

Answer ____________________

MP1 **14.** A sports club is having a beginner's triathlon race. To finish the race, each athlete must swim $1\frac{5}{10}$ miles, run $3\frac{1}{10}$ miles, and ride a bicycle for $5\frac{8}{10}$ miles. What is the total length of the race in miles?

Show your work.

Answer ____________________

Independent Practice

MP2 **15.** There are 12 eggs in 1 carton. In the delivery truck, $4\frac{5}{12}$ cartons of eggs break. In the store, a total of $2\frac{9}{12}$ cartons break. How many more cartons of eggs break in the truck than in the store?

Show your work.

Answer ____________________

MP6 **16.** Justin subtracts $5\frac{1}{3}$ from $10\frac{2}{3}$ using the method shown at the right. Is Justin's difference correct?

$$\begin{array}{r} 10\frac{2}{3} \\ -\ 5\frac{1}{3} \\ \hline 5\frac{1}{3} \end{array}$$

Answer ____________________

Justify your answer using words, drawings, or numbers.

MP8 **17.** Mariah needs to add $4\frac{10}{100}$ and $3\frac{20}{100}$. She starts by using a denominator of 10 to write equivalent fractions for $\frac{10}{100}$ and $\frac{20}{100}$. Then she adds the mixed numbers. Can Mariah find the correct sum with this method?

Answer ____________________

Justify your answer using words, drawings, or numbers.

Lesson

20 Problem Solving: Add and Subtract Fractions

Essential Question:
How can modeling a fraction word problem help you understand it?
4.NF.3d

Guided Instruction

In this lesson you will learn how to solve addition and subtraction problems involving fractions and mixed numbers with like denominators.

Understand: Using fraction models to represent and solve problems

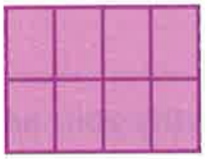
Sarah has a watercolor paint set with 8 colors. First she uses up her favorite $\frac{1}{8}$ of the colors. When she uses up another $\frac{5}{8}$ of the colors, she buys a new paint set. How much of the paint set is left when Sarah buys a new one?

To solve the problem, you can make a model to represent the problem.

First read for information.

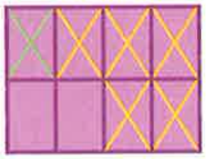
Sarah uses up $\frac{1}{8}$ and $\frac{5}{8}$ of the set. The denominator 8 in $\frac{1}{8}$ and $\frac{5}{8}$ shows the whole is divided into eighths.

Draw a model to represent the whole as $\frac{8}{8}$.



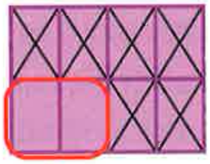
1 paint set = $\frac{8}{8}$

Use different color Xs to show the $\frac{1}{8}$ and $\frac{5}{8}$ that Sarah uses.



Then read again to find the question.
The problem asks how much of the paint set is left, not how much Sarah uses altogether.
So you need to subtract the sum of $\frac{1}{8} + \frac{5}{8}$ from the whole.

$\frac{8}{8} - (\frac{1}{8} + \frac{5}{8}) = \frac{2}{8}$



$\frac{2}{8}$ left

When Sarah buys a new one, $\frac{2}{8}$ of the paint set is left. Notice that the model shows the same answer.

What if the problem asks how many of the 8 colors are left? Explain how to use your model to find this number.

Guided Instruction

Understand: Using equations to represent and solve problems

The library is $2\frac{5}{6}$ miles from the school. David's father picks him up from school to go to the library. On the way, they stop at a store. Then they walk $1\frac{4}{6}$ miles to the library. How far is the store from the school?

You can use a model to help write an equation.

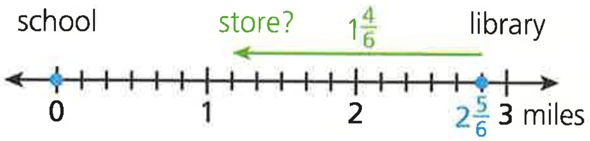
Identify what you know.

- The distance between the school and the library is $2\frac{5}{6}$ miles.
- The store is between the school and library.
- The distance between the store and library is $1\frac{4}{6}$ miles.

Identify what you need to find.

- the distance from the store to the school

Draw a model to represent the problem.
The store is $1\frac{4}{6}$ miles away from the library.
This location on the number line is the distance from the store to the school.



Write an equation for the problem.
You can use d to represent the unknown distance.

$$d + 1\frac{4}{6} = 2\frac{5}{6}$$

$$d = 2\frac{5}{6} - 1\frac{4}{6}$$

Subtract $1\frac{4}{6}$ from $2\frac{5}{6}$ to find d.

Since $\frac{4}{6} < \frac{5}{6}$, you can subtract without regrouping the 2 wholes.

Remember! Check whether you can subtract the whole numbers and the fractions separately.

school to library − store to library = school to store

$$2\frac{5}{6} \quad - \quad 1\frac{4}{6} \quad = \quad 1\frac{1}{6}$$

$$d = 1\frac{1}{6}$$

The store is $1\frac{1}{6}$ miles from the school.

Guided Instruction

Connect: What you know about writing equivalent fractions and using models.

Ava has $2\frac{4}{10}$ pounds of frozen strawberries. She needs $\frac{2}{10}$ pound of strawberries to make 1 shake. How many strawberry shakes can Ava make with all $2\frac{4}{10}$ pounds?

To solve the problem, use what you know about equivalent fractions to write an equation.

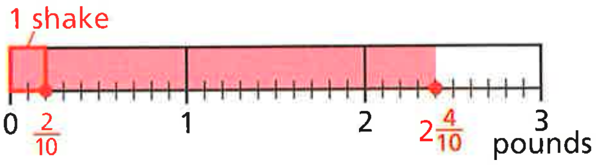
Step 1

Identify and model what you know.

- The total amount is $2\frac{4}{10}$ pounds.
- The amount for 1 shake is $\frac{2}{10}$ pounds.

Identify what you need to find.

- how many shakes Ava can make

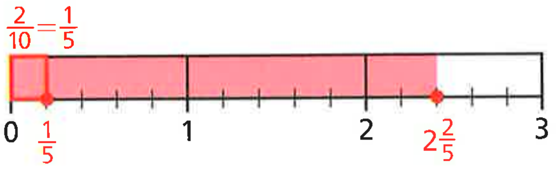

1 shake
0
$\frac{2}{10}$
1
2
$2\frac{4}{10}$
3
pounds

Step 2

Using a unit fraction makes it easier to find the number of shakes Ava can make. Find the unit fraction that is equivalent to $\frac{2}{10}$.

$$\frac{2}{10} = \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$$

Ava needs $\frac{1}{5}$ pound of strawberries to make each shake.


$\frac{2}{10} = \frac{1}{5}$
0
$\frac{1}{5}$
1
2
$2\frac{2}{5}$
3

Step 3

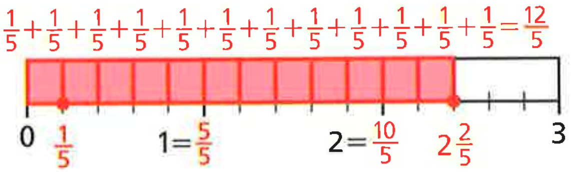
Plan how you will find the number of fifths in $2\frac{2}{5}$. You can count the unit fractions of $\frac{1}{5}$ on the model, or you can write $2\frac{2}{5}$ as a sum of unit fractions, or you can write $2\frac{2}{5}$ as a fraction to find the total number of fifths.

Remember!
The numerator in a fraction shows the number of equal parts in a fraction.

Step 4

$$2\frac{2}{5} = 1 + 1 + \frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = \frac{12}{5}$$

There are 12 fifths in $2\frac{2}{5}$.

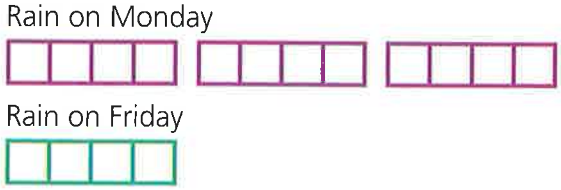

$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{12}{5}$
0
$\frac{1}{5}$
$1=\frac{5}{5}$
$2=\frac{10}{5}$
$2\frac{2}{5}$
3

Ava can make 12 strawberry shakes with $2\frac{4}{10}$ pounds of strawberries.

Guided Practice

Use the model to answer the questions.

1. A town gets $2\frac{3}{4}$ inches of rain on Monday and $\frac{2}{4}$ inches of rain on Friday. How much rain does the town get on those two days?



a. Color the model to show the given information.

b. Circle the equation you can use to solve the problem.

$2\frac{3}{4} - \frac{2}{4} = r$ $\quad$ $2\frac{3}{4} + \frac{2}{4} = r$ $\quad$ $2\frac{3}{4} + r = \frac{2}{4}$

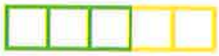
c. Solve the problem. Show your work.

$2\frac{3}{4} + \frac{2}{4} =$ ________

Answer ____________________

Write an addition or subtraction equation to represent the problem. You can use the model to help. Then solve.

2. Vincent has a bag of apples. Green apples fill $\frac{3}{5}$ of the bag. Yellow apples fill the rest of the bag. What fraction of the bag has yellow apples?



a. Equation ____________

b. Solve the problem. Show your work.

Answer ____________________

Think • Pair • Share

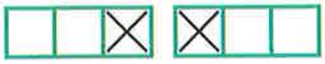
MP1 **3.** What are the whole amounts in exercises 1 and 2? How do you know that the mixed numbers and the fractions in each exercise are for the same whole? Explain.

Independent Practice

Use the model to represent the problem. Write and solve an equation to find the answer.

1. Brad has $1\frac{1}{3}$ boxes of birthday candles. He uses $\frac{2}{3}$ of the birthday candles on a birthday cake. What fraction of the candles is left?

a. Equation ____________



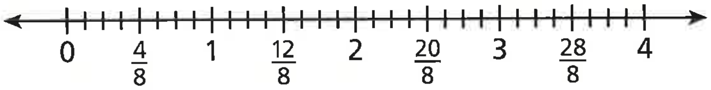
b. Solve the problem. Show your work.

Answer ____________

2. Kate's favorite song is $3\frac{5}{8}$ minutes long. Ty's favorite song is $2\frac{7}{8}$ minutes long. How much longer is Kate's favorite song than Ty's?

a. Equation

____



b. Solve the problem. Show your work.

Answer ____________

Circle the correct answer.

3. Ayden works on his homework for $1\frac{1}{6}$ hours before dinner. It takes him another $1\frac{3}{6}$ hours to finish his homework after dinner. How much time does Ayden spend on homework?

a. $\frac{2}{6}$ hour

b. $1\frac{2}{6}$ hours

c. $1\frac{4}{6}$ hours

d. $2\frac{4}{6}$ hours

4. Talia is $\frac{1}{4}$ of her mother's age. Carl is $\frac{1}{4}$ of his mother's age. Who is older, Talia or Carl? Which statement about the problem is true?

a. Subtract $\frac{1}{4} - \frac{1}{4}$ to solve.

b. Add $\frac{1}{4} + \frac{1}{4}$ to solve.

c. Carl and Talia must be the same age, since they are the same fraction of their mothers' ages.

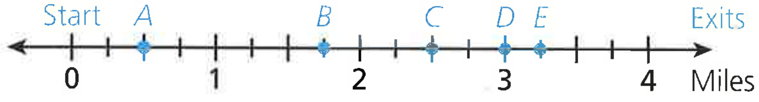
d. The problem cannot be solved without the ages of Carl and Talia's mothers.

Independent Practice

For exercises 5–8, use the number line. For exercises 6–9, write and solve an equation.

A car gets on the highway at the starting point, 0 miles. The next five highway exits are shown on the number line below.

Distances Between Highway Exits



5. Label Exits *A*, *B*, *C*, and *E* with the fraction or mixed number for that location.

6. What is the distance between Exits *A* and *B*?

Equation ____________

Answer ______________________________

7. A driver misses Exit *B* and has to get off at Exit *C*. How far is Exit *C* from Exit *B*?

Equation ____________

Answer ______________________________

8. Mauricio and his mother get on the highway at Exit *A* and stay on it until Exit *E*. For how many miles do Mauricio and his mother stay on the highway?

Equation ____________

Answer ______________________________

9. Lauren's family is going to an amusement park. The drive usually takes $2\frac{2}{5}$ hours. Today it takes an extra $\frac{4}{5}$ hour. How long does it take Lauren's family today to drive to the amusement park?

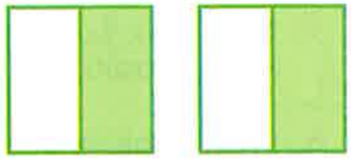
Equation ____________

Answer ______________________________

Independent Practice

MP6 **10.** Dennis has a jigsaw puzzle that is missing $\frac{2}{100}$ of the edge pieces and $\frac{4}{100}$ of the inside pieces. Does this mean that the puzzle is only missing 6 pieces altogether?

MP3 **11.** Tracy says that the sum of the fractions modeled at the right could be either 1 or $\frac{2}{4}$. Do you agree? Explain.



Solve the problems.

MP2 **12.** Two of the 10 players on Ryan's lacrosse team are sick. Another player has to miss practice to go to the dentist. What fraction of Ryan's team is missing from practice?

Show your work.

Answer ______________________________

MP1 **13.** An acre is an area that is a little smaller than a football field. Jeremy says that the size of the school playground is about $\frac{5}{12}$ of an acre. Susan says that the playground is at least $1\frac{6}{12}$ acres bigger than Jeremy thinks it is. About what does Susan think the area of the playground is?

Show your work.

Answer ______________________________

Independent Practice

MP6 **14.** The southern elephant seal at the zoo weighs $3\frac{1}{8}$ tons. The biggest hippopotamus at the zoo weighs $1\frac{4}{8}$ tons. How much more does the elephant seal weigh than the hippopotamus?

Show your work.

Answer ____________________

MP1 **15.** Jessica's bookcase is $4\frac{9}{12}$ feet tall. She wants to replace it with a taller bookcase that is $6\frac{1}{12}$ feet tall. Jessica says the new bookcase is about 1 foot taller. Is this a reasonable estimate?

Show your work.

Answer ____________________

MP2 **16.** Billy is estimating the combined amount of milk in two jugs. He says that $\frac{1}{2}$ gallon plus $\frac{4}{6}$ gallon must be greater than 1 whole gallon. Do you agree?

Answer ____________________

Justify your answer using words, drawings, or numbers.

MP7 **17.** Damon says that to write $1\frac{63}{100}$ as a sum of unit fractions, you need 163 addends. Kenya says that you need 64 addends. Who is correct?

Answer ____________________

Justify your answer using words, drawings, or numbers.

Lesson

21 Multiply Unit Fractions by Whole Numbers

Essential Question:
How is multiplying a unit fraction by a whole number like multiplying a whole number by a whole number?
4.NF.4a

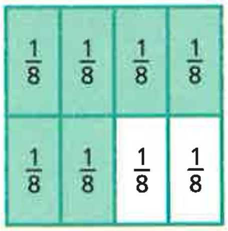
Guided Instruction

In this lesson you will learn how to multiply a unit fraction by a whole number and how to write any fraction as a product of a whole number and a unit fraction.

Understand: Multiplying a unit fraction by a whole number

There are 8 school photos on a whole sheet. So, each photo is $\frac{1}{8}$ of the sheet. Gary gives 6 of his school photos to friends. What fraction of a whole sheet does Gary give to his friends?

To find the fraction of a whole sheet made up of 6 photos that Gary gives to his friends you can use addition or multiplication.



The model shows that 6 photos are $\frac{6}{8}$ of a whole sheet.

Each photo is $\frac{1}{8}$ of a sheet. There are 6 photos.

$$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{1+1+1+1+1+1}{8}=\frac{6}{8}$$

Since all the addends are the same, you can think of them as equal groups.

$$\overbrace{\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}^{6\text{ addends}}=6\times\frac{1}{8}$$

So, $6\times\frac{1}{8}=\frac{6}{8}$

and $\frac{6}{8}=6\times\frac{1}{8}$.

Gary gives $\frac{6}{8}$ of a whole sheet of photos to his friends.

Remember!
To find the total of equal groups, multiply the number in each group by the number of groups.

Since $\frac{6}{8}$ is the product $6\times\frac{1}{8}$, $\frac{6}{8}$ is a multiple of $\frac{1}{8}$.

Remember!
A multiple of a number is a product of that number and a whole number.

Count by eighths to write the first five multiples of $\frac{1}{8}$.

Connect: Writing a mixed number or a fraction greater than 1 as a product of a whole number and a unit fraction

Luz runs $1\frac{2}{5}$ miles around the park on Saturday. If one lap around the park is a distance of $\frac{1}{5}$ mile, how many laps around the park does Luz run in $1\frac{2}{5}$ miles?

To find the number of laps, write $1\frac{2}{5}$ as a product of a whole number and $\frac{1}{5}$.

$$1\frac{2}{5} = \overbrace{n}^{\text{number of laps}} \times \frac{1}{5}$$

Step 1

Rewrite $1\frac{2}{5}$ as a fraction greater than 1.

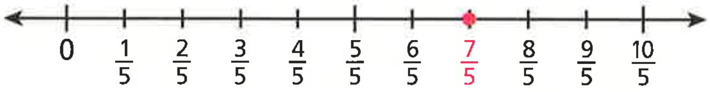
$1\frac{2}{5} = 1 + \frac{2}{5} = \frac{5}{5} + \frac{2}{5} = \frac{7}{5}$

Step 2

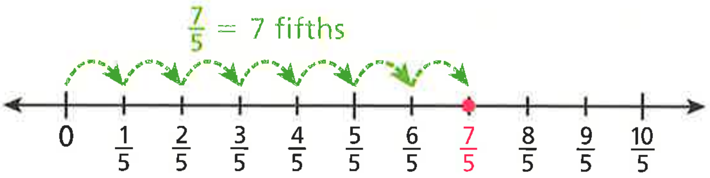
Rewrite the equation $1\frac{2}{5} = n \times \frac{1}{5}$ as $\frac{7}{5} = n \times \frac{1}{5}$.

Step 3

Find n, the number of laps. You can use a number line as a model.
The number line shows the first ten multiples of $\frac{1}{5}$.
Label $\frac{7}{5}$ on the number line.



Now, use the number line to find the number of fifths in $\frac{7}{5}$.



There are 7 fifths in $\frac{7}{5}$.
So, $\frac{7}{5} = 7 \times \frac{1}{5}$.
$n = 7$.

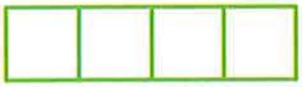
➡ Luz runs 7 laps around the park in $1\frac{2}{5}$ miles.

Write $\frac{7}{5}$ as a sum of unit fractions. How many addends are there?

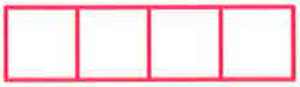
Guided Practice

Complete the equation to find each product. Color the model to show the product.

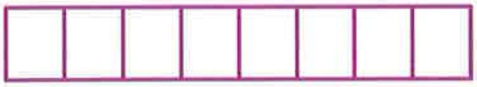
1. $2 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} = \frac{}{4}$



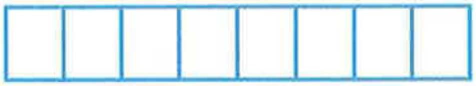
2. $3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{}{4}$



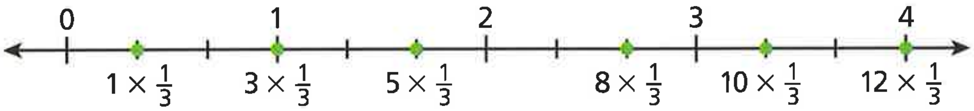
3. $4 \times \frac{1}{8} = \frac{}{8}$



4. $5 \times \frac{1}{8} = \frac{}{}$



For exercises 5–10, write the multiple of $\frac{1}{3}$ that names each point on the number line.


0
1
2
3
4
$1 \times \frac{1}{3}$
$3 \times \frac{1}{3}$
$5 \times \frac{1}{3}$
$8 \times \frac{1}{3}$
$10 \times \frac{1}{3}$
$12 \times \frac{1}{3}$

5. $1 \times \frac{1}{3} =$ ____

6. $3 \times \frac{1}{3} =$ ____

7. $5 \times \frac{1}{3} =$ ____

8. $8 \times \frac{1}{3} =$ ____

9. $10 \times \frac{1}{3} =$ ____

10. $12 \times \frac{1}{3} =$ ____

11. Write the missing fractions to count by thirds.

$\frac{1}{3}$, ____, $\frac{3}{3}$, ____, $\frac{5}{3}$, ____, ____, $\frac{8}{3}$, ____, $\frac{10}{3}$, ____, $\frac{12}{3}$

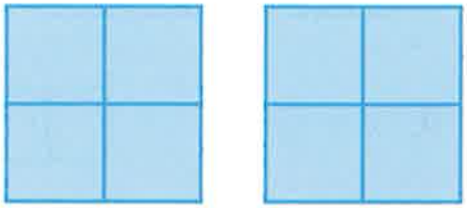
Guided Practice

Solve the problems.

12. Complete the number pattern of multiples.
Rule: Start with $\frac{1}{2}$ and add $\frac{1}{2}$.

$\frac{1}{2}$, ____, $\frac{3}{2}$, ____, ____, ____, ____, ____, . . .

13. Sam has 2 pounds of peanuts. He is going to make snack bags with $\frac{1}{4}$ pound of peanuts each. How many snack bags can Sam make? Explain your answer.



Write each fraction or mixed number as the product of a whole number and a unit fraction.

14. $\frac{2}{5} = 2 \times$ __________

15. $\frac{4}{6} =$ __________

16. $\frac{10}{8} =$ __________

17. $\frac{7}{10} =$ __________

18. $1\frac{2}{3} =$ __________

19. $2\frac{2}{5} =$ __________

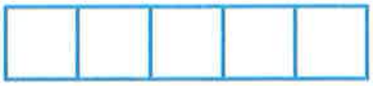
Think • Pair • Share

MP3 **20.** Explain why the denominator stays the same when you multiply a unit fraction by a whole number.

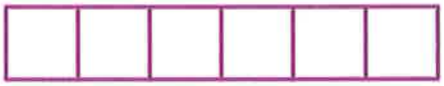
Independent Practice

Complete the equation to find each product. Color the model to support your answer.

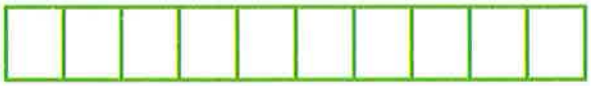
1. $4 \times \frac{1}{5} = \frac{\;}{5}$



2. $1 \times \frac{1}{6} = \frac{\;}{\;}$

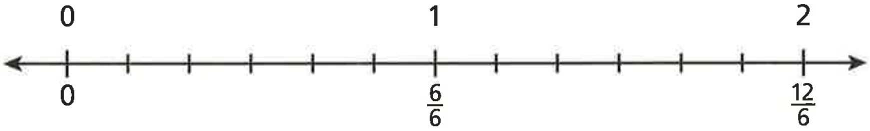


3. $10 \times \frac{1}{10} = \frac{\;}{\;}$

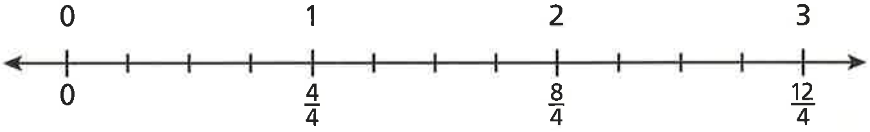


Complete the equation to find each product. Use the number line model to support your answer.

4. $3 \times \frac{1}{6} = \frac{\;}{6}$



5. $5 \times \frac{1}{4} = \frac{\;}{\;}$



Draw lines to match.

6. $2 \times \frac{1}{6}$

7. $2 \times \frac{1}{10}$

8. $10 \times \frac{1}{2}$

9. $12 \times \frac{1}{10}$

10. $12 \times \frac{1}{2}$

a. $\frac{12}{10}$

b. $\frac{10}{2}$

c. $\frac{2}{10}$

d. $\frac{6}{2}$

e. $\frac{12}{2}$

f. $\frac{2}{6}$

Independent Practice

Write each fraction or mixed number as the product of a whole number and a unit fraction.

11. $\frac{5}{6} = 5 \times$ __________

12. $\frac{4}{4} =$ __________

13. $\frac{7}{12} =$ __________

14. $\frac{20}{3} =$ __________

15. $1\frac{5}{10} =$ __________

16. $3\frac{1}{3} =$ __________

Solve.

17. Circle all the multiples of $\frac{1}{5}$.

$\frac{2}{5}$ $\frac{1}{10}$ $\frac{3}{5}$ $\frac{3}{4}$ $\frac{7}{6}$ $\frac{11}{5}$

a. Write each multiple of $\frac{1}{5}$ as a product of a whole number and $\frac{1}{5}$.

b. Write each multiple of $\frac{1}{5}$ as a sum.

Circle the correct answer or answers.

18. Which of the sums are also multiples of $\frac{1}{3}$?

a. $\frac{1}{1} + \frac{1}{2} + \frac{1}{3}$

b. $1 + \frac{1}{3}$

c. $\frac{1}{2} + \frac{1}{1}$

d. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

19. Which fraction is a multiple of $\frac{1}{8}$ that equals 1?

a. $\frac{1}{8}$

b. $\frac{8}{8}$

c. $\frac{8}{1}$

d. $\frac{80}{10}$

Independent Practice

MP3 **20.** Carrie says that fractions can only have other fractions as multiples. Eva says that this is not true, because 2 is a multiple of $\frac{1}{2}$. Who is correct? Explain.

MP3 **21.** Nicolas and Jun Ho are making up riddles for multiples of fractions. Nicolas says his number is a multiple of $\frac{1}{3}$ that has an even number as a denominator. Jun Ho says this is impossible. Can you guess Nicolas's number? Explain.

Solve the problems.

MP1 **22.** A restaurant makes big hamburgers with $\frac{1}{4}$ pound of beef in each. If the restaurant makes 12 hamburgers in one hour, how many pounds of beef does the restaurant use?

Show your work.

Answer ______________________________

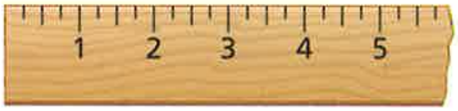
MP4 **23.** Reagan's town has a clock tower with bells that ring once every $\frac{1}{2}$ hour. If the bells ring 6 times, how many hours have gone by?

Show your work.

Answer ______________________________

Independent Practice

MP5 **24.** The picture shows a ruler marked with some fractions of an inch. The markings are multiples of $\frac{1}{4}$ and $\frac{1}{2}$.



For each of these fractions, what is the greatest multiple less than 5 inches? To help you identify each multiple, write it as a product of the fraction and a whole number.

Show your work.

Answer ______________________________

MP4 **25.** Victoria says that $\frac{17}{6}$ is not a multiple of $\frac{1}{6}$. As proof, she says that $\frac{17}{6}$ equals $2\frac{5}{6}$ instead of a whole number. Is Victoria correct? Draw a model to justify your answer.

Answer ______________________________

Justify your answer using words, drawings, or numbers.

MP8 **26.** Samir says that you can write any fraction as a product of a whole number and a unit fraction. Do you agree? Give an example to support your answer.

Answer ______________________________

Justify your answer using words, drawings, or numbers.

Lesson

22 Multiply Fractions by Whole Numbers

Essential Question:
How can you multiply a fraction that is not a unit fraction by a whole number?
4.NF.4b

Guided Instruction

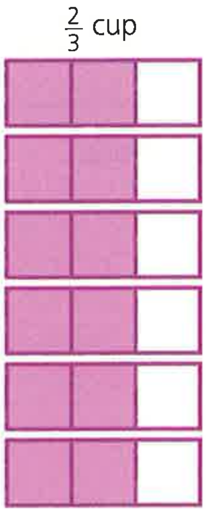
In this lesson you will learn how to multiply any fraction by a whole number.

Understand: Multiplication as repeated addition

Sabrina is making carrot muffins. Her muffin pan has 6 baking cups. Sabrina will put $\frac{2}{3}$ cup of batter in each baking cup. How much batter does she need for all 6 baking cups?

To find the total cups of batter, multiply $6 \times \frac{2}{3}$.

Draw a model for the 6 groups of $\frac{2}{3}$.
The model shows that $6 \times \frac{2}{3}$ is the same as $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$.



$$6 \times \frac{2}{3} = \overbrace{\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}}^{6 \text{ addends}}$$

$$= \frac{2 + 2 + 2 + 2 + 2 + 2}{3}$$

$$= \frac{6 \times 2}{3}$$

$$= \frac{12}{3}$$

Write the product $\frac{12}{3}$ as a whole number. You can use the fact $12 = 4 \times 3$ to help you find the whole number.

$$\frac{12}{3} = \frac{4 \times 3}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} = 4$$

➡ Sabrina needs 4 cups of batter for all 6 baking cups.

Remember!
You can break apart the numerator of a fraction in the same way as a whole number.

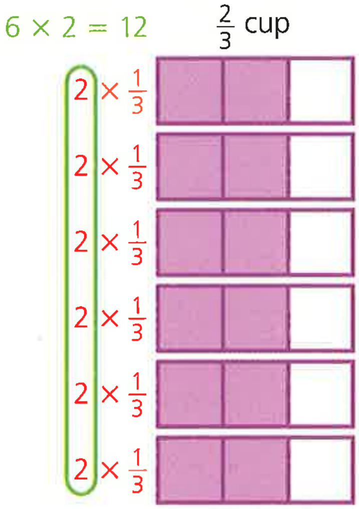
Write the first six multiples of $\frac{2}{3}$.

Guided Instruction

Understand: Writing the fraction as a multiple of a unit fraction before multiplying

How can you multiply a fraction by a whole number without using repeated addition?

You can solve Sabrina's muffin problem in a different way. To multiply $6 \times \frac{2}{3}$, you can think of $\frac{2}{3}$ as $2 \times \frac{1}{3}$ and then use the Associative Property of Multiplication.



$$\begin{aligned} 6 \times \frac{2}{3} &= 6 \times (\overbrace{2 \times \frac{1}{3}}^{\text{2 thirds}}) \\ &= (6 \times 2) \times \frac{1}{3} \\ &= 12 \times \frac{1}{3} \\ &= \frac{12}{3} \\ &= 4 \end{aligned}$$

Remember! The Associative Property of Multiplication states that you can change the grouping of the factors without changing the product.

The product is the same as with the repeated addition method.

➡ To multiply any fraction by a whole number, you can multiply the numerator of the fraction by the whole number. Then write the product over the denominator.

$$6 \times \frac{2}{3} = \frac{6 \times 2}{3} = \frac{12}{3} = 4$$

- Find the total cups of batter Sabrina will need for a pan with 9 baking cups instead of 6.

Guided Instruction

Connect: Multiplying a mixed number by a whole number

Damian is teaching himself to play the guitar. This week, he plays his guitar for $1\frac{3}{4}$ hours on each of 4 days. For how many hours does Damian play his guitar this week?

To find how many hours Damian plays his guitar, multiply $4 \times 1\frac{3}{4}$.
You can use an equivalent fraction for the mixed number.

Step 1

Rewrite $1\frac{3}{4}$ as a fraction greater than 1.

$$\begin{aligned} 1\frac{3}{4} &= 1 + \frac{3}{4} \\ &= \frac{4}{4} + \frac{3}{4} \\ &= \frac{7}{4} \end{aligned}$$

So, rewrite the multiplication as $4 \times \frac{7}{4}$.

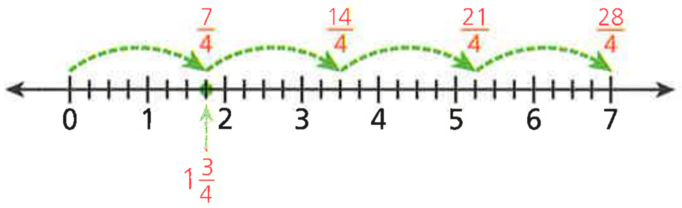
Step 2

Multiply 4 times the numerator in the fraction $\frac{7}{4}$.
Write the total number of fourths over the denominator.

$$4 \times \frac{7}{4} = \frac{4 \times 7}{4} = \frac{28}{4} = 7$$

To check, you can mark off 4 groups of $\frac{7}{4}$ on a number line.

The fourth multiple of $\frac{7}{4}$ is the product, $\frac{28}{4} = 7$.



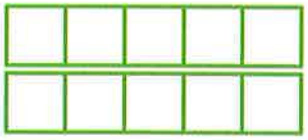
Damian plays his guitar for 7 hours this week.

Sarah says that you can also multiply $4 \times 1\frac{3}{4}$ by writing $4 \times (1 + \frac{3}{4})$ and then adding 4×1 and $4 \times \frac{3}{4}$. Do you agree?

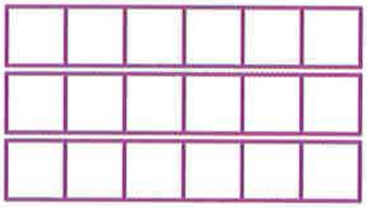
Guided Practice

Use the model or the number line to show that your product is correct.

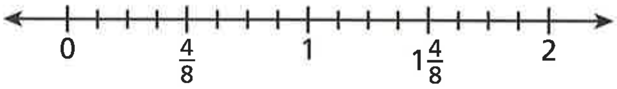
1. $2 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} = \frac{\underline{\quad} \times 3}{5} = \frac{\underline{\quad}}{\underline{\quad}} = 1\frac{\underline{\quad}}{\underline{\quad}}$



2. $3 \times \frac{4}{6} = \frac{4}{6} + \frac{4}{6} + \frac{4}{6} = \frac{\underline{\quad} \times \underline{\quad}}{\underline{\quad}} = \frac{\underline{\quad}}{\underline{\quad}} = \underline{\qquad}$



3. $3 \times \frac{5}{8} = \frac{\underline{\quad} \times \underline{\quad}}{8} = \frac{\underline{\quad}}{8} = \underline{\qquad}\frac{\underline{\quad}}{\underline{\quad}}$



Complete the equation to multiply. If the product is greater than 1, write it as a whole or mixed number.

4. $7 \times \frac{3}{10} = \frac{\underline{\quad} \times \underline{\quad}}{10} = \frac{\underline{\quad}}{10} = \underline{\qquad}\frac{\underline{\quad}}{\underline{\quad}}$

5. $4 \times 2\frac{1}{2} = 4 \times \frac{\underline{\quad}}{2} = \frac{\underline{\quad} \times \underline{\quad}}{\underline{\quad}} = \frac{\underline{\quad}}{\underline{\quad}} = \underline{\qquad}$

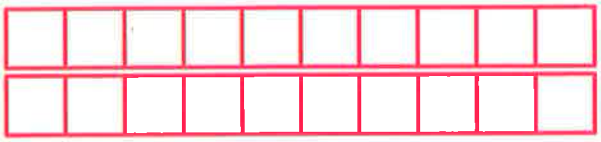
Think • Pair • Share

MP4 **6.** Compare the problem in exercise 2 to multiplying the whole numbers 3 × 4. How are the problems alike? How are they different?

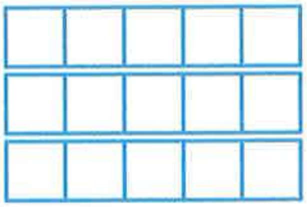
Independent Practice

Use the model or the number line to show that your product is correct.

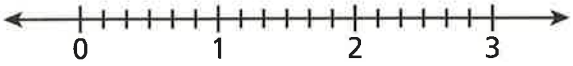
1. $2 \times \frac{7}{10} = \frac{\underline{\quad} \times \underline{\quad}}{10} = \frac{\quad}{\quad} = 1\frac{\quad}{\quad}$



2. $3 \times \frac{4}{5} = \frac{\underline{\quad} \times \underline{\quad}}{\quad} = \frac{\quad}{\quad} = \underline{\qquad}\frac{\quad}{\quad}$



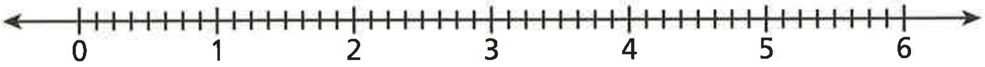
3. $5 \times \frac{3}{6} = \frac{\underline{\quad} \times \underline{\quad}}{\quad} = \frac{\quad}{\quad} = \underline{\qquad}\frac{\quad}{\quad}$



List the first five multiples of $\frac{3}{6}$. Use the model to help you.

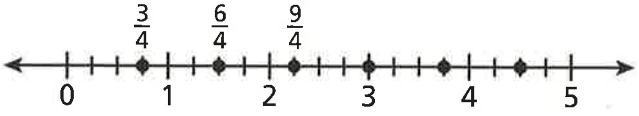
____

4. $4 \times 1\frac{3}{8} = 4 \times \frac{\quad}{8} = $



Independent Practice

For exercises 5–9, use the number line. The number line shows multiples of $\frac{3}{4}$. Write each multiple as a product of a whole number and a fraction.



5. $\frac{3}{4} =$ ____________

6. $\frac{6}{4} =$ ____________

7. $\frac{9}{4} =$ ____________

8. What are the next three multiples?

____

9. Are there any multiples of $\frac{3}{4}$ between $\frac{6}{4}$ and $\frac{9}{4}$? Explain.

Multiply. If the product is greater than 1, write it as a whole or mixed number. You can use a model to help.

10. $5 \times \frac{2}{5} =$ ____________

11. $4 \times \frac{5}{6} =$ ____________

12. $3 \times \frac{9}{12} =$ ____________

13. $6 \times 2\frac{2}{10} =$ ____________

Independent Practice

MP2 **14.** Brenda wants to write the product $\frac{14}{3}$ as a mixed number. How can Brenda use multiplication to help her rename the product?

MP1 **15.** How does drawing a model help you multiply a fraction by a whole number? Explain your opinion.

Solve the problems.

MP1 **16.** Kristin is making soup. The recipe calls for $\frac{6}{8}$ pound of beef. Kristin wants to make 4 times the amount of soup in the recipe. How many pounds of beef does Kristin need?

Show your work.

Answer ______________________________

MP6 **17.** Levi watched his favorite movie 9 times on DVD. The movie is $1\frac{6}{10}$ hours long. How many hours did it take Levi to watch the movie 9 times?

Show your work.

Answer ______________________________

Independent Practice

MP1 **18.** A gardener is moving some houseplants into bigger pots. She has 3 plastic pots that hold $4\frac{1}{12}$ quarts of soil each. She also has 4 clay pots that hold $3\frac{2}{5}$ quarts of soil each. Does the gardener need more soil for the plastic pots or the clay pots?

Show your work.

Answer ________________________________

MP1 **19.** Darius reads $\frac{2}{10}$ of his book in 3 days. He says that if he reads the rest of the book at the same speed, it should take him 15 more days to finish the book. Is Darius correct?

Answer ________________________________

Justify your answer using words, drawings, or numbers.

MP7 **20.** Jenny is helping to pack supplies for a school picnic. Each of three boxes of drinking straws are filled with only 50 out of 100 straws. Jenny says that she can multiply $3 \times \frac{1}{2}$ instead of $3 \times \frac{50}{100}$ to find the total number of full boxes. Is Jenny correct?

Answer ________________________________

Justify your answer using words, drawings, or numbers.

Lesson

23 Problem Solving: Multiply Fractions by Whole Numbers

Essential Question:
How can multiplying fractions by whole numbers help you to solve real-world problems?
4.NF.4c

Guided Instruction

In this lesson you will learn how to multiply a fraction or a mixed number by a whole number to solve word problems.

Understand: Multiplying a fraction by a whole number and comparing two amounts

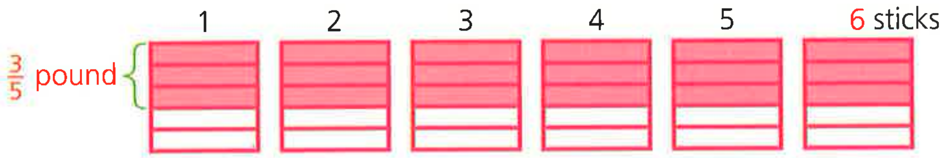
Jordan is buying modeling clay to use for a project. He can buy six $\frac{3}{5}$-pound sticks of different colors. The store also sells a 5-pound pack of colors for the same price. Which should Jordan buy to get more clay?

To solve the problem, find the total weight of the 6 sticks and then compare it to 5 pounds.

Remember!

$6 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$

You can use a model to show 6 groups of $\frac{3}{5}$ pound.



Find the total number of pounds in 6 sticks. You can count the shaded fifths in the model or you can write and solve an equation. Use w to represent the number of pounds of clay in 6 sticks.

$w = 6 \times \frac{3}{5}$

$6 \times \frac{3}{5} = \frac{6 \times 3}{5} = \frac{18}{5}$ and $\frac{18}{5} = \frac{15}{5} + \frac{3}{5} = 3 + \frac{3}{5} = 3\frac{3}{5}$

$w = 3\frac{3}{5}$

The total weight of the 6 sticks is $3\frac{3}{5}$ pounds.

Compare $3\frac{3}{5}$ to 5. Since $3 < 5$, the 5-pound pack has more clay. $3\frac{3}{5} < 5$

Jordan should buy the 5-pound pack instead of six $\frac{3}{5}$-pound sticks to get more clay.

How many fifths of a pound are in the 5-pound pack? How many more pounds are in the 5-pound pack than in six $\frac{3}{5}$-pound sticks?

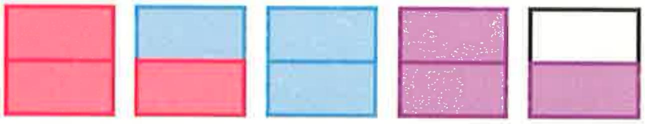
Guided Instruction

Understand: Multiplying a fraction or mixed number greater than 1 by a whole number

Jordan buys 3 different colors of paint. He needs $1\frac{1}{2}$ gallons of each color. How much paint does he buy?

Method 1

Solve the problem using visual models. Show 3 groups of $1\frac{1}{2}$.



The model shows that Jordan buys $\frac{9}{2}$ or $4\frac{1}{2}$ gallons of paint.

➡ Jordan buys $4\frac{1}{2}$ gallons of paint.

Method 2

Solve the problem using an equation. Let g represent the total number of gallons of paint Jordan buys. $g = 3 \times 1\frac{1}{2}$

One Way to Solve the Equation

$g = 3 \times 1\frac{1}{2}$

$g = 3 \times (1 + \frac{1}{2})$

$g = 3 \times (\frac{2}{2} + \frac{1}{2})$

$g = 3 \times (\frac{3}{2})$

$g = \frac{3 \times 3}{2}$

$g = \frac{9}{2}$

$g = \frac{8}{2} + \frac{1}{2}$

$g = 4 + \frac{1}{2}$

$g = 4\frac{1}{2}$

Another Way to Solve the Equation

$g = 3 \times 1\frac{1}{2}$

$g = 3 \times (1 + \frac{1}{2})$

$g = (3 \times 1) + (3 \times \frac{1}{2})$

$g = 3 + \frac{3}{2}$

$g = 3 + (\frac{2}{2} + \frac{1}{2})$

$g = 3 + (1 + \frac{1}{2})$

$g = 3 + 1\frac{1}{2}$

$g = 4\frac{1}{2}$

Remember!

Multiplying a number by a sum is the same as multiplying the number by each addend of the sum and then adding the products.

➡ Jordan buys $4\frac{1}{2}$ gallons of paint.

If Jordan can only buy gallon cans of paint, how many gallon cans must Jordan buy? Explain your reasoning.

Lesson 23 Problem Solving: **Multiply Fractions by Whole Numbers**

Guided Instruction

Connect: Solving a multi-step problem using equations and models

Natalie and Rachel are hiking with their families. Natalie's family hikes a $3\frac{1}{4}$-mile trail to the end, and then hikes back on the same trail. Rachel's family hikes around a $\frac{3}{4}$-mile loop 8 times. Which family hikes more miles?

You can use equations and models to solve.

Step 1

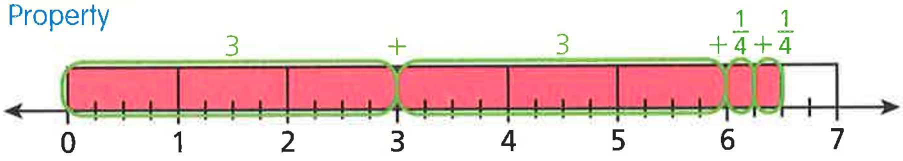
Remember!
Estimate: $2 \times 3 = 6$

Find the total distance Natalie's family hikes.

Use n for the number of miles Natalie's family hikes.

Solve $n = 2 \times 3\frac{1}{4}$.

$$n = 2 \times 3\frac{1}{4} = 2 \times \left(3 + \frac{1}{4}\right)$$
$$= (2 \times 3) + \left(2 \times \frac{1}{4}\right) \leftarrow \text{Distributive Property}$$
$$= 6 + \frac{2 \times 1}{4}$$
$$= 6\frac{2}{4}$$



So, $n = 6\frac{2}{4}$. Natalie's family hikes $6\frac{2}{4}$ miles.

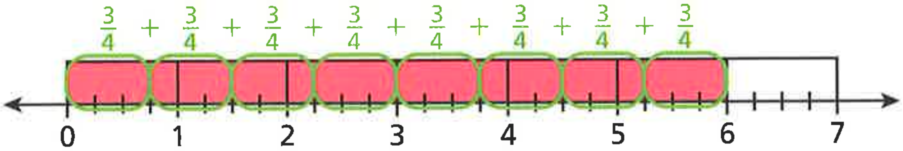
Step 2

Find the total distance Rachel's family hikes.

Use r for the number of miles Rachel's family hikes.

Solve $r = 8 \times \frac{3}{4}$.

$$r = 8 \times \frac{3}{4} = \frac{8 \times 3}{4}$$
$$= \frac{24}{4}$$
$$= 6$$


$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$
0
1
2
3
4
5
6
7

So, $r = 6$. Rachel's family hikes 6 miles.

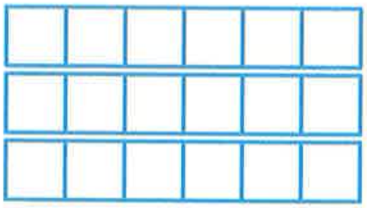
Step 3

Compare the distances: $6\frac{2}{4} > 6$.

➡ Natalie's family hikes more miles than Rachel's family.

Guided Practice

Use the model to represent the word problem. Answer the questions below.



1. The class learns about 3 different subjects for $\frac{5}{6}$ hour each. What is the total amount of time the class spends on these 3 subjects?

a. What do you need to find? ______________________

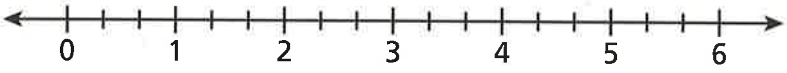
b. Color the model to show the given information.

c. Write an equation you can use to solve the problem. Use h to represent the unknown number of hours. ____________

d. Multiply to solve the equation. $h = 3 \times \frac{5}{6} = \frac{\_\_ \times \_\_}{6} = \_\_$

Answer ______________________

2. Lee needs 4 pieces of ribbon, each $1\frac{1}{3}$ yards long. How many yards of ribbon does she need in all?



a. What do you need to find? ______________________

b. Draw and label ____ loops on the number line to represent the problem.

c. Write an equation that you can use to solve the problem. Use r to represent the unknown. ____________

d. Solve the equation. If the solution is equal to or greater than 1, write it as a whole or mixed number.

$r =$ ______

Answer ______________________

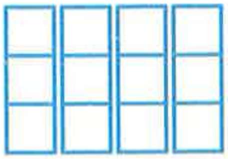
Think • Pair • Share

MP2 **3.** Show a different way to multiply $4 \times 1\frac{1}{3}$. Which way do you prefer?

Independent Practice

For problems 1–3, identify what you need to find. Then use the model and an equation to solve the problem.

1. Alyssa has 4 snack bags of raisins. Each bag has $\frac{2}{3}$ cup of raisins. How many cups of raisins does Alyssa have altogether?

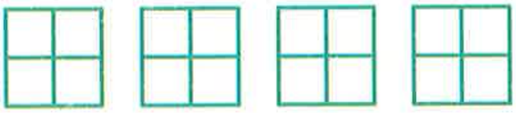


Find ______________________________

Equation ______________________________

Answer ______________________________

2. Luis mixes equal amounts of orange, cranberry, and grape juice to make a fruit punch. He uses $1\frac{1}{4}$ cups of each kind of juice. How many cups of punch does Luis make?

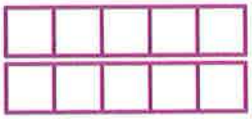


Find ______________________________

Equation ______________________________

Answer ______________________________

MP5 3. In Diana's class, $\frac{2}{5}$ of the students ride the Blue Bus to school. In Troy's class, 2 times as many students ride the Blue Bus as in Diana's class. If both classes have the same number of students, what fraction of Troy's class rides the Blue Bus?



Find ______________________________

Equation ______________________________

Answer ______________________________

MP1 4. If 12 students in Diana's class ride the Blue Bus, how many students in Troy's class ride the Blue Bus? Use exercise 3 to solve.

Show your work.

Answer ______________________________